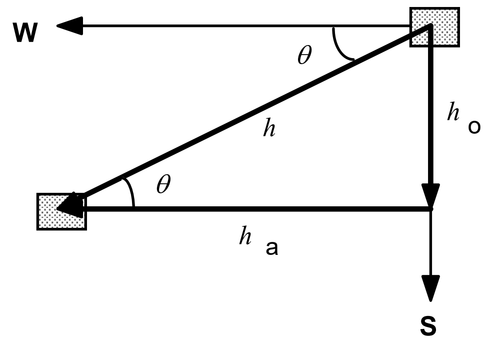
13. SSM ***REASONING*** The shortest distance between the two towns is along the line that joins them. This distance, h, is the hypotenuse of a right triangle whose other sides are h_o = 35.0 km and h_a = 72.0 km, as shown in the figure below.



SOLUTION The angle θ is given by $\tan\theta = h_o / h_a$ so that

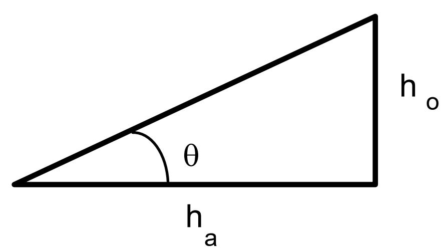
$$\theta = \tan^{-1}\left(\frac{35.0\text{ km}}{72.0\text{ km}}\right) = \boxed{25.9^\circ \text{ S of W}}$$

We can then use the Pythagorean theorem to find h.

$$h = \sqrt{h_o^2 + h_a^2} = \sqrt{(35.0\text{ km})^2 + (72.0\text{ km})^2} = \boxed{80.1\text{ km}}$$

---

14. ***REASONING*** The drawing shows a schematic representation of the hill. We know that the hill rises 12.0 m vertically for every 100.0 m of distance in the horizontal direction, so that $h_o = 12.0$ m and $h_a = 100.0$ m. Moreover, according to Equation 1.3, the tangent function is $\tan\theta = h_o / h_a$. Thus, we can use the inverse tangent function to determine the angle θ.



SOLUTION With the aid of the inverse tangent function (see Equation 1.6) we find that

$$\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right) = \tan^{-1}\left(\frac{12.0\text{ m}}{100.0\text{ m}}\right) = \boxed{6.84^\circ}$$

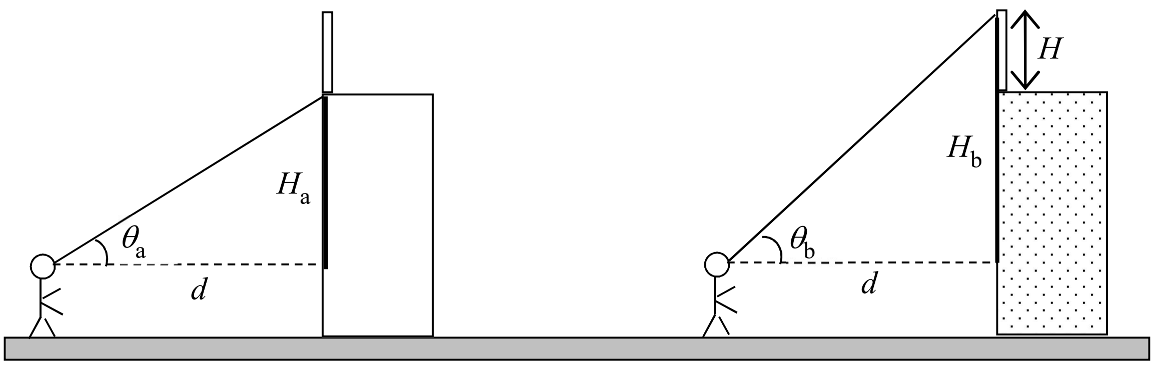
---

16. ***REASONING*** In both parts of the drawing the line of sight, the horizontal dashed line, and the vertical form a right triangle. The angles θ_a = 35.0° and θ_b = 38.0° at which the person's line of sight rises above the horizontal are known, as is the horizontal distance d = 85.0 m from the building. The unknown vertical sides of the right triangles correspond, respectively, to the heights H_a and H_b of the bottom and top of the antenna relative to the person's eyes. The antenna's height H is the *difference* between H_b and H_a: $H = H_b - H_a$. The horizontal side d of the triangle is adjacent to the angles θ_a and θ_b, while the vertical sides H_a and H_b are

opposite these angles. Thus, in either triangle, the angle θ is related to the horizontal and vertical sides by Equation 1.3 $\left(\tan\theta = \frac{h_o}{h_a}\right)$:

$$\tan\theta_a = \frac{H_a}{d} \quad (1)$$

$$\tan\theta_b = \frac{H_b}{d} \quad (2)$$



(*a*) (*b*)

SOLUTION Solving Equations (1) and (2) for the heights of the bottom and top of the antenna relative to the person's eyes, we find that

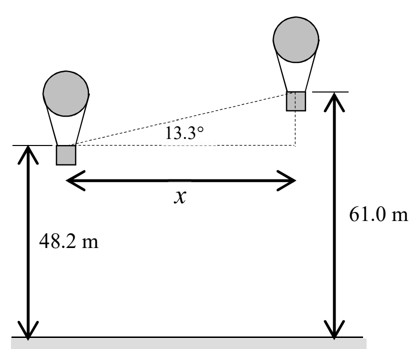
$$H_a = d\tan\theta_a \qquad \text{and} \qquad H_b = d\tan\theta_b$$

The height of the antenna is the difference between these two values:

$$H = H_b - H_a = d\tan\theta_b - d\tan\theta_a = d\left(\tan\theta_b - \tan\theta_a\right)$$
$$H = (85.0\text{ m})\left(\tan 38.0^\circ - \tan 35.0^\circ\right) = \boxed{6.9\text{ m}}$$

---

17. ***REASONING*** The drawing shows the heights of the two balloonists and the horizontal distance *x* between them. Also shown in dashed lines is a right triangle, one angle of which is 13.3°. Note that the side adjacent to the 13.3° angle is the horizontal distance *x*, while the side opposite the angle is the distance between the two heights, 61.0 m – 48.2 m. Since we know the angle and the length of one side of the right triangle, we can use trigonometry to find the length of the other side.



SOLUTION The definition of the tangent function, Equation 1.3, can be used to find the horizontal distance *x*, since the angle and the length of the opposite side are known:

$$\tan 13.3° = \frac{\text{length of opposite side}}{\text{length of adjacent side } (= x)}$$

Solving for *x* gives

$$x = \frac{\text{length of opposite side}}{\tan 13.3°} = \frac{61.0 \text{ m} - 48.2 \text{ m}}{\tan 13.3°} = \boxed{54.1 \text{ m}}$$

24. ***REASONING*** Since the initial force and the resultant force point along the east/west line, the second force must also point along the east/west line. The direction of the second force is not specified; it could point either due east or due west, so there are two answers. We use "N" to denote the units of the forces, which are specified in newtons.

SOLUTION If the second force points $\boxed{\text{due east}}$, both forces point in the same direction and the magnitude of the resultant force is the sum of the two magnitudes: $F_1 + F_2 = F_R$. Therefore,

$$F_2 = F_R - F_1 = 400 \text{ N} - 200 \text{ N} = \boxed{200 \text{ N}}$$

If the second force points $\boxed{\text{due west}}$, the two forces point in opposite directions, and the magnitude of the resultant force is the difference of the two magnitudes: $F_2 - F_1 = F_R$. Therefore,

$$F_2 = F_R + F_1 = 400 \text{ N} + 200 \text{ N} = \boxed{600 \text{ N}}$$

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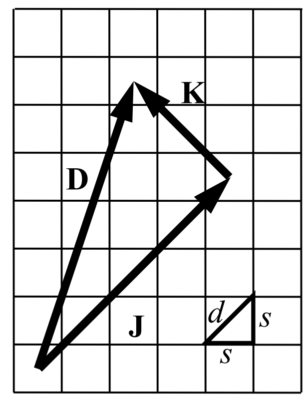
25. $\boxed{\textbf{SSM}}$ ***REASONING*** For convenience, we can assign due east to be the positive direction and due west to be the negative direction. Since all the vectors point along the same east-west line, the vectors can be added just like the usual algebraic addition of positive and negative scalars. We will carry out the addition for all of the possible choices for the two vectors and identify the resultants with the smallest and largest magnitudes.

SOLUTION There are six possible choices for the two vectors, leading to the following resultant vectors:



$$\mathbf{F_1} + \mathbf{F_2} = 50.0 \text{ newtons} + 10.0 \text{ newtons} = +60.0 \text{ newtons} = 60.0 \text{ newtons, due east}$$

$$\mathbf{F_1} + \mathbf{F_3} = 50.0 \text{ newtons} - 40.0 \text{ newtons} = +10.0 \text{ newtons} = 10.0 \text{ newtons, due east}$$

$$\mathbf{F_1} + \mathbf{F_4} = 50.0 \text{ newtons} - 30.0 \text{ newtons} = +20.0 \text{ newtons} = 20.0 \text{ newtons, due east}$$

$$\mathbf{F_2} + \mathbf{F_3} = 10.0 \text{ newtons} - 40.0 \text{ newtons} = -30.0 \text{ newtons} = 30.0 \text{ newtons, due west}$$

$$\mathbf{F_2} + \mathbf{F_4} = 10.0 \text{ newtons} - 30.0 \text{ newtons} = -20.0 \text{ newtons} = 20.0 \text{ newtons, due west}$$

$$\mathbf{F_3} + \mathbf{F_4} = -40.0 \text{ newtons} - 30.0 \text{ newtons} = -70.0 \text{ newtons} = 70.0 \text{ newtons, due west}$$

The resultant vector with the smallest magnitude is $\boxed{\mathbf{F_1} + \mathbf{F_3} = 10.0 \text{ newtons, due east}}$.

The resultant vector with the largest magnitude is $\boxed{\mathbf{F_3} + \mathbf{F_4} = 70.0 \text{ newtons, due west}}$.

28. ***REASONING*** The triple jump consists of a double jump in one direction, followed by a perpendicular single jump, which we can represent with displacement vectors **J** and **K** (see the drawing). These two perpendicular vectors form a right triangle with their resultant **D** = **J** + **K**, which is the displacement of the colored checker. In order to find the magnitude *D* of the displacement, we first need to find the magnitudes *J* and *K* of the double jump and the single jump. As the three sides of a right triangle, *J*, *K*, and *D* (the hypotenuse) are related to one another by the Pythagorean theorem (Equation 1.7) The double jump moves the colored checker a straight-line distance equal to the length of four square's diagonals *d*, and the single jump moves a length equal to two square's diagonals. Therefore,

$$J = 4d \qquad \text{and} \qquad K = 2d \tag{1}$$

Let the length of a square's side be s. Any two adjacent sides of a square form a right triangle with the square's diagonal (see the drawing). The Pythagorean theorem gives the diagonal length d in terms of the side length s:

$$d = \sqrt{s^2 + s^2} = \sqrt{2s^2} = s\sqrt{2} \tag{2}$$

SOLUTION First, we apply the Pythagorean theorem to the right triangle formed by the three displacement vectors, using Equations (1) for J and K:

$$D = \sqrt{J^2 + K^2} = \sqrt{(4d)^2 + (2d)^2} = \sqrt{16d^2 + 4d^2} = \sqrt{20d^2} = d\sqrt{20} \tag{3}$$

Substituting Equation (2) into Equation (3) gives

$$D = d\sqrt{20} = \left(s\sqrt{2}\right)\sqrt{20} = s\sqrt{40} = (4.0\text{ cm})\sqrt{40} = \boxed{25\text{ cm}}$$

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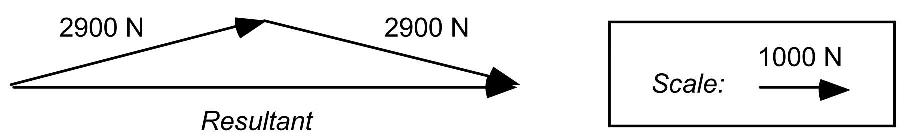
31. SSM ***REASONING AND SOLUTION*** The single force needed to produce the same effect is equal to the resultant of the forces provided by the two ropes. The following figure shows the force vectors drawn to scale and arranged tail to head. The magnitude and direction of the resultant can be found by direct measurement using the scale factor shown in the figure.



a. From the figure, the magnitude of the resultant is $\boxed{5600\text{ N}}$.

b. The single rope should be directed $\boxed{\text{along the dashed line}}$ in the text drawing.

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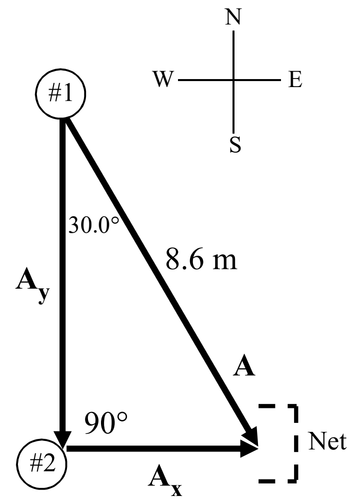
36. ***REASONING*** The triangle in the drawing is a right triangle. We know one of its angles is 30.0°, and the length of the hypotenuse is 8.6 m. Therefore, the sine and cosine functions can be used to find the magnitudes of $\mathbf{A_x}$ and $\mathbf{A_y}$. The directions of these vectors can be found by examining the diagram.

SOLUTION

a. The magnitude A_x of the displacement vector $\mathbf{A_x}$ is related to the length of the hypotenuse and the 30.0° angle by the sine function (Equation 1.1). The drawing shows that the direction of $\mathbf{A_x}$ is due east.



$$A_x = A\sin 30.0° = (8.6\text{ m})\sin 30.0° = \boxed{4.3\text{ m, due east}}$$

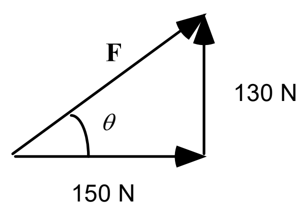
b. In a similar manner, the magnitude A_y of $\mathbf{A_y}$ can be found by using the cosine function (Equation 1.2). Its direction is due south.

$$A_y = A\cos 30.0° = (8.6\text{ m})\cos 30.0° = \boxed{7.4\text{ m, due south}}$$

40. ***REASONING AND SOLUTION***

a. From the Pythagorean theorem, we have

$$F = \sqrt{(150\text{ N})^2 + (130\text{ N})^2} = \boxed{2.0 \times 10^2\text{ N}}$$



b. The angle θ is given by

$$\theta = \tan^{-1}\left(\frac{130\text{ N}}{150\text{ N}}\right) = \boxed{41°}$$

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46. ***REASONING*** To apply the component method for vector addition, we must first determine the x and y components of each vector. The algebraic sum of the three x components gives the x component of the resultant. The algebraic sum of the three y components gives the y component of the resultant. Knowing the x and y components of the resultant will allow us to use the Pythagorean theorem to determine the magnitude of the resultant. Finally, the directional angle of the resultant will be obtained using the trigonometric sine function.

SOLUTION Referring to the drawing in the text, we see that the x and y components of the vectors are

$$A_x = -A\cos 20.0° = -(5.00\text{ m})\cos 20.0° = -4.70\text{ m}$$

$$B_x = B\cos 60.0° = (5.00\text{ m})\cos 60.0° = 2.50\text{ m}$$

$$C_x = 0.00\text{ m}$$

$$R_x = -4.70\text{ m} + 2.50\text{ m} + 0.00\text{ m} = -2.20\text{ m}$$

$$A_y = A\sin 20.0° = (5.00\text{ m})\sin 20.0° = 1.71\text{ m}$$

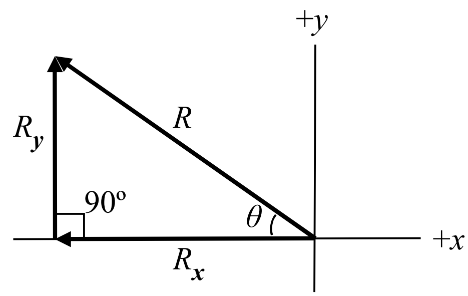
$$B_y = B\sin 60.0° = (5.00\text{ m})\sin 60.0° = 4.33\text{ m}$$

$$C_y = -4.00\text{ m}$$

$$R_y = 1.71\text{ m} + 4.33\text{ m} - 4.00\text{ m} = 2.04\text{ m}$$

Note that the value for A_x is negative because this component points in the $-x$ direction and that the value for C_x is zero because the vector **C** points along the $-y$ axis. Note also that the value for C_y is negative because the vector **C** points along the $-y$ axis.

The x component R_x of the resultant vector, being negative, points in the $-x$ direction. The y component R_y of the resultant vector, being positive, points in the $+y$ direction. The drawing shows these two components and the resultant vector. Since the components are perpendicular, the magnitude R of the resultant can be obtained using the Pythagorean theorem.



$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-2.20\text{ m})^2 + (2.04\text{ m})^2} = \boxed{3.00\text{ m}}$$

Referring to the drawing, we can see that $\sin\theta = R_x / R$, so that the directional angle θ is

$$\theta = \sin^{-1}\left(\frac{R_y}{R}\right) = \sin^{-1}\left(\frac{2.04\text{ m}}{3.00\text{ m}}\right) = 42.8°$$

Thus, the resultant vector points in a direction of $\boxed{42.8° \text{ above the negative } x \text{ axis}}$.

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47. ***REASONING*** Using the component method for vector addition, we will find the x component of the resultant force vector by adding the x components of the individual vectors. Then we will find the y component of the resultant vector by adding the y components of the individual vectors. Once the x and y components of the resultant are known, we will use the Pythagorean theorem to find the magnitude of the resultant and trigonometry to find its direction. We will take east as the $+x$ direction and north as the $+y$ direction.

SOLUTION The x component of the resultant force **F** is

$$F_x = \underbrace{(2240\text{ N})\cos 34.0°}_{F_{Ax}} + \underbrace{(3160\text{ N})\cos 90.0°}_{F_{Bx}} = (2240\text{ N})\cos 34.0°$$

The y component of the resultant force **F** is

$$F_y = \underbrace{-(2240\text{ N})\sin 34.0°}_{F_{Ay}} + \underbrace{(-3160\text{ N})}_{F_{By}}$$

Using the Pythagorean theorem, we find that the magnitude of the resultant force is

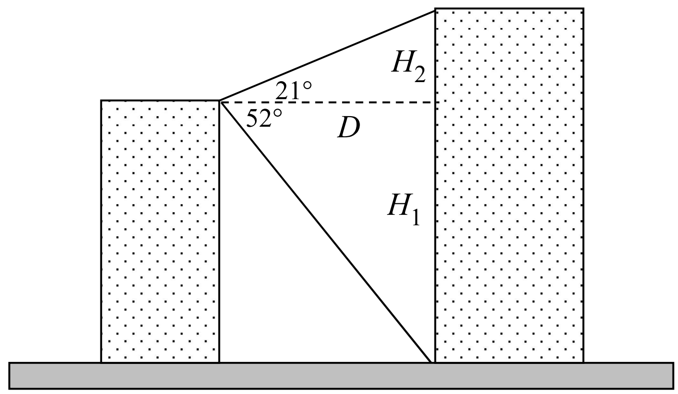
$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{\left[(2240\text{ N})\cos 34.0°\right]^2 + \left[-(2240\text{ N})\sin 34.0° - 3160\text{ N}\right]^2} = \boxed{4790\text{ N}}$$

Using trigonometry, we find that the direction of the resultant force is

$$\theta = \tan^{-1}\left[\frac{(2240\text{ N})\sin 34.0° + 3160\text{ N}}{(2240\text{ N})\cos 34.0°}\right] = \boxed{67.2° \text{ south of east}}$$

---

68. ***REASONING*** There are two right triangles in the drawing. Each contains the common side that is shown as a dashed line and is labeled D, which is the distance between the buildings. The hypotenuse of each triangle is one of the lines of sight to the top and base of the taller building. The remaining (vertical) sides of the triangles are labeled H_1 and H_2.



Since the height of the taller building is $H_1 + H_2$ and the height of the shorter building is H_1, the ratio that we seek is $(H_1 + H_2)/H_1$. We will use the tangent function to express H_1 in terms of the 52° angle and to express H_2 in terms of the 21° angle. The unknown distance D will be eliminated algebraically when the ratio $(H_1 + H_2)/H_1$ is calculated.

SOLUTION The ratio of the building heights is

$$\frac{\text{Height of taller building}}{\text{Height of shorter building}} = \frac{H_1 + H_2}{H_1}$$

Using the tangent function, we have that

$$\tan 52° = \frac{H_1}{D} \quad \text{or} \quad H_1 = D\tan 52°$$

$$\tan 21° = \frac{H_2}{D} \quad \text{or} \quad H_2 = D\tan 21°$$

Substituting these results into the expression for the ratio of the heights gives

$$\frac{\text{Height of taller building}}{\text{Height of shorter building}} = \frac{H_1 + H_2}{H_1} = \frac{D\tan 52° + D\tan 21°}{D\tan 52°}$$

$$= 1 + \frac{\tan 21°}{\tan 52°} = \boxed{1.30}$$

Since 1.30 is less than 1.50, $\boxed{\text{your friend is wrong}}$.

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69. ***REASONING AND SOLUTION*** If **D** is the unknown vector, then **A** + **B** + **C** + **D** = 0 requires that $D_E = -(A_E + B_E + C_E)$ or

$$D_E = (113\text{ u})\cos 60.0° - (222\text{ u})\cos 35.0° - (177\text{ u})\cos 23.0° = \boxed{-288\text{ units}}$$

The minus sign indicates that D_E has a direction of due west.

Also, $D_N = -(A_N + B_N + C_N)$ or

$$D_N = (113\text{ u})\sin 60.0° + (222\text{ u})\sin 35.0° - (177\text{ u})\sin 23.0° = \boxed{156\text{ units}}$$
