Chapter 3

Kinematics in Two Dimensions

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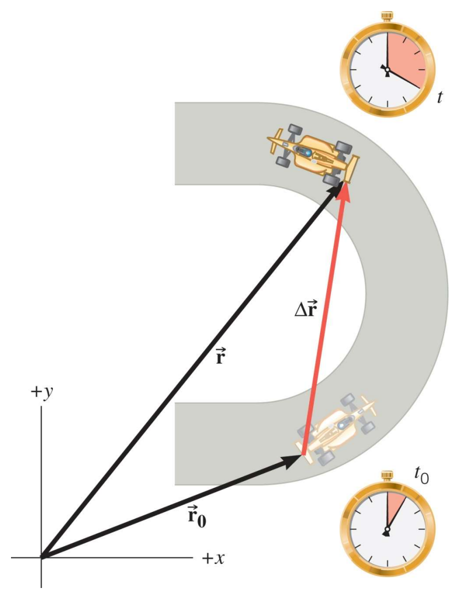
3.1 Displacement, Velocity, and Acceleration

$\vec{\mathbf{r}}_o$ = initial position

$\vec{\mathbf{r}}$ = final position

$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}} - \vec{\mathbf{r}}_o$ = displacement



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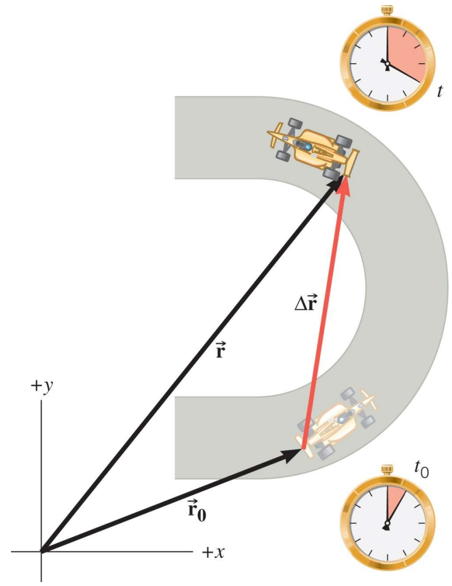
3.1 *Displacement, Velocity, and Acceleration*

Average velocity is the displacement divided by the elapsed time.

$$\bar{\vec{\mathbf{v}}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$



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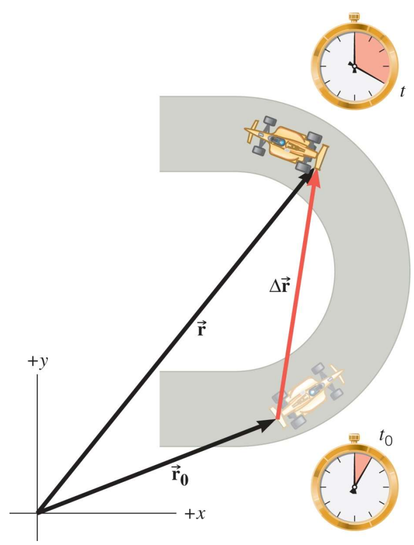
3.1 *Displacement, Velocity, and Acceleration*

The ***instantaneous velocity*** indicates how fast the car moves and the direction of motion at each instant of time.

$$\bar{\vec{\mathbf{v}}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$



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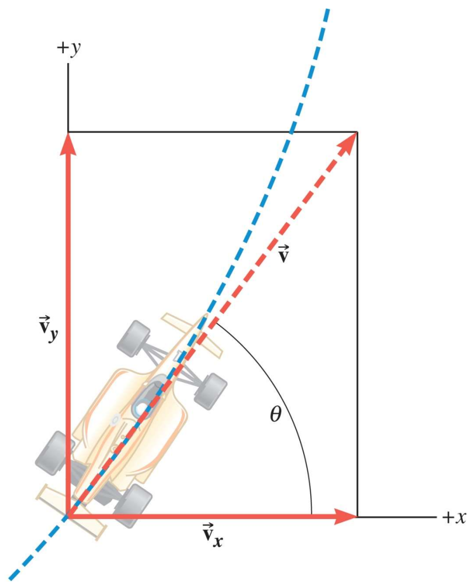
3.1 Displacement, Velocity, and Acceleration

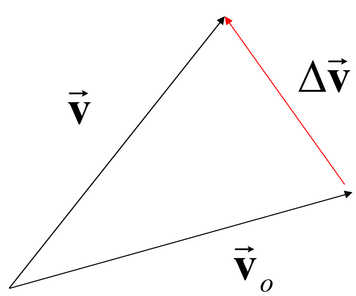
$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$



3.1 Displacement, Velocity, and Acceleration

DEFINITION OF AVERAGE ACCELERATION

$$\overline{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o} = \frac{\Delta \vec{\mathbf{v}}}{\Delta t}$$



3.2 Equations of Kinematics in Two Dimensions

Equations of Kinematics

$$v = v_o + at$$

$$x = \tfrac{1}{2}\left(v_o + v\right)t$$

$$v^2 = v_o^2 + 2ax$$

$$x = v_o t + \tfrac{1}{2}at^2$$

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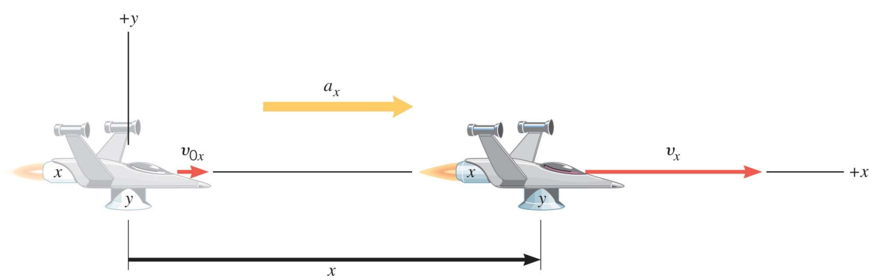
3.2 Equations of Kinematics in Two Dimensions



$$v_x = v_{ox} + a_x t$$

$$x = \tfrac{1}{2}\left(v_{ox} + v_x\right)t$$

$$x = v_{ox}t + \tfrac{1}{2}a_x t^2$$

$$v_x^2 = v_{ox}^2 + 2a_x x$$

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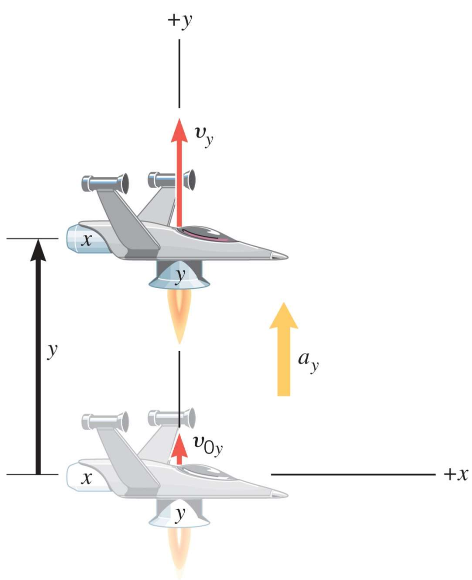
3.2 Equations of Kinematics in Two Dimensions



$$v_y = v_{oy} + a_y t$$

$$y = v_{oy} t + \tfrac{1}{2} a_y t^2$$

$$y = \tfrac{1}{2}\left(v_{oy} + v_y\right) t$$

$$v_y^2 = v_{oy}^2 + 2a_y y$$

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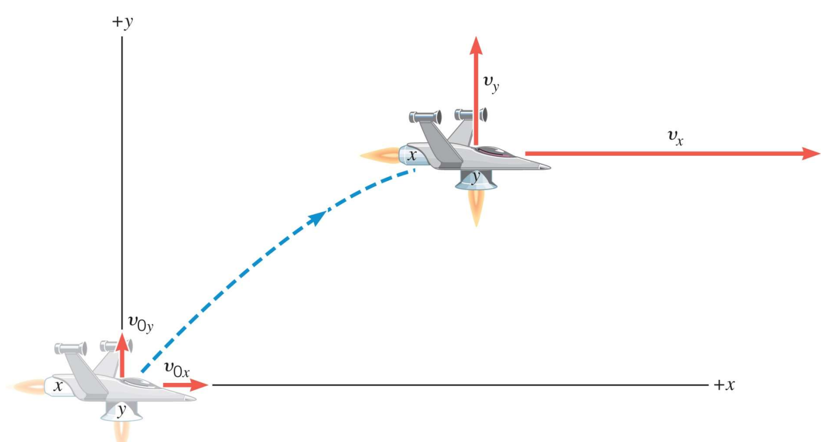
3.2 Equations of Kinematics in Two Dimensions



The x part of the motion occurs exactly as it would if the y part did not occur at all, and vice versa.

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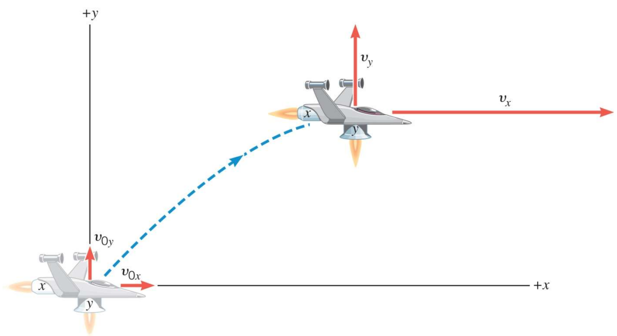
3.2 *Equations of Kinematics in Two Dimensions*

Example 1 A Moving Spacecraft

In the *x* direction, the spacecraft has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. At a time 7.0 s, find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of the spacecraft.



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3.2 *Equations of Kinematics in Two Dimensions*

Reasoning Strategy

1. Make a drawing.

2. Decide which directions are to be called positive (+) and negative (-).

3. Write down the values that are given for any of the five kinematic variables associated with each direction.

4. Verify that the information contains values for at least three of the kinematic variables. Do this for *x* and *y*. Select the appropriate equation.

5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next.

6. Keep in mind that there may be two possible answers to a kinematics problem.

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3.2 Equations of Kinematics in Two Dimensions

Example 1 A Moving Spacecraft

In the *x* direction, the spacecraft has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22 m/s	7.0 s

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14 m/s	7.0 s

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3.2 Equations of Kinematics in Two Dimensions

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22 m/s	7.0 s

$$x = v_{ox}t + \tfrac{1}{2}a_x t^2$$
$$= (22\,\text{m/s})(7.0\,\text{s}) + \tfrac{1}{2}(24\,\text{m/s}^2)(7.0\,\text{s})^2 = +740\,\text{m}$$

$$v_x = v_{ox} + a_x t$$
$$= (22\,\text{m/s}) + (24\,\text{m/s}^2)(7.0\,\text{s}) = +190\,\text{m/s}$$

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3.2 *Equations of Kinematics in Two Dimensions*

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14 m/s	7.0 s

$$y = v_{oy}t + \tfrac{1}{2}a_y t^2$$

$$= (14\,\text{m/s})(7.0\,\text{s}) + \tfrac{1}{2}(12\,\text{m/s}^2)(7.0\,\text{s})^2 = +390\,\text{m}$$

$$v_y = v_{oy} + a_y t$$

$$= (14\,\text{m/s}) + (12\,\text{m/s}^2)(7.0\,\text{s}) = +98\,\text{m/s}$$

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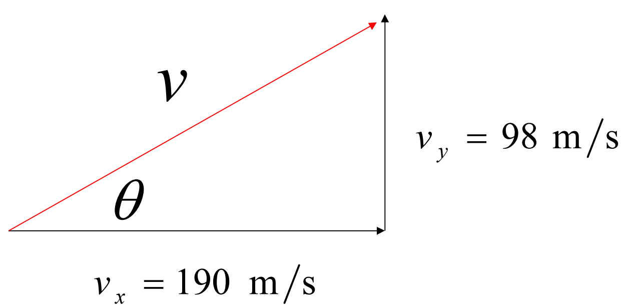
3.2 *Equations of Kinematics in Two Dimensions*



$$v = \sqrt{(190\ \text{m/s})^2 + (98\ \text{m/s})^2} = 210\ \text{m/s}$$

$$\theta = \tan^{-1}(98/190) = 27^\circ$$

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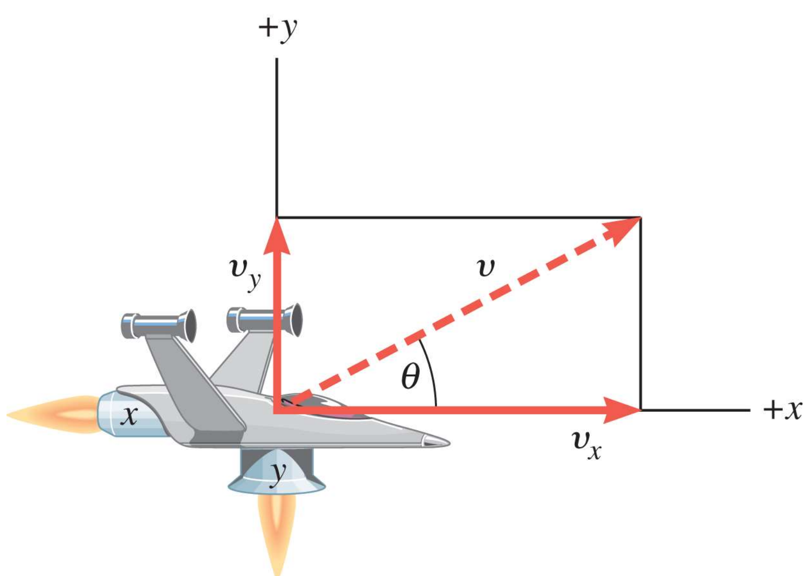
3.2 Equations of Kinematics in Two Dimensions



3.3 Projectile Motion

Under the influence of gravity alone, an object near the surface of the Earth will accelerate downwards at $9.80 m/s^2$.

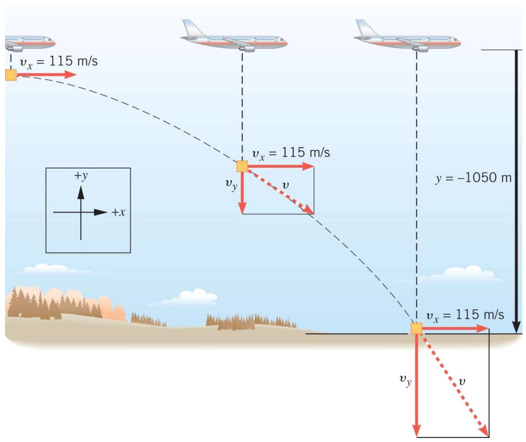
$$a_y = -9.80\,\mathrm{m/s^2} \qquad a_x = 0$$

$$\rightarrow \quad v_x = v_{ox} = \text{constant}$$

3.3 ***Projectile Motion***

Example 3 A Falling Care Package

The airplane is moving horizontally with a constant velocity of +115 m/s at an altitude of 1050m. Determine the time required for the care package to hit the ground.



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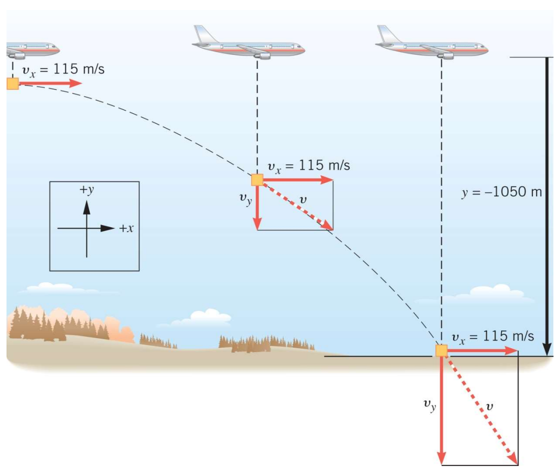
3.3 ***Projectile Motion***



y	a_y	v_y	v_{oy}	t
-1050 m	-9.80 m/s^2		0 m/s	?

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3.3 ***Projectile Motion***

y	a_y	v_y	v_{oy}	t
-1050 m	-9.80 m/s^2		0 m/s	?

$$y = v_{oy}t + \tfrac{1}{2}a_y t^2 \quad \Longrightarrow \quad y = \tfrac{1}{2}a_y t^2$$

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-1050\,\text{m})}{-9.80\,\text{m/s}^2}} = 14.6\,\text{s}$$

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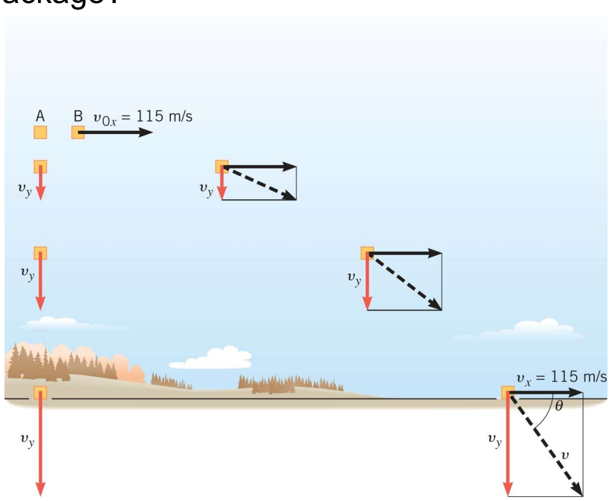
3.3 ***Projectile Motion***

Example 4 The Velocity of the Care Package

What are the magnitude and direction of the final velocity of the care package?



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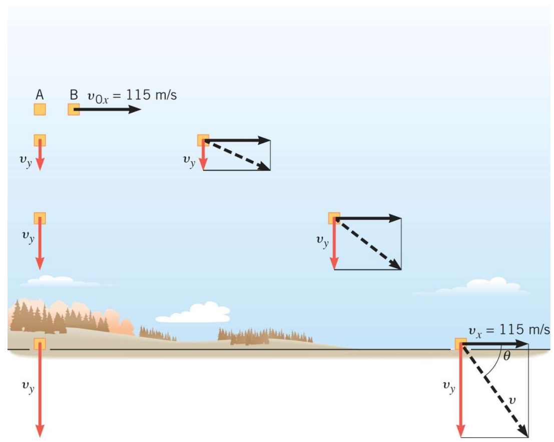
3.3 *Projectile Motion*



y	a_y	v_y	v_{oy}	t
-1050 m	-9.80 m/s^2	?	0 m/s	14.6 s

3.3 *Projectile Motion*

y	a_y	v_y	v_{oy}	t
-1050 m	-9.80 m/s^2	?	0 m/s	14.6 s

$$v_y = v_{oy} + a_y t = 0 + \left(-9.80\,\mathrm{m/s^2}\right)\left(14.6\,\mathrm{s}\right)$$

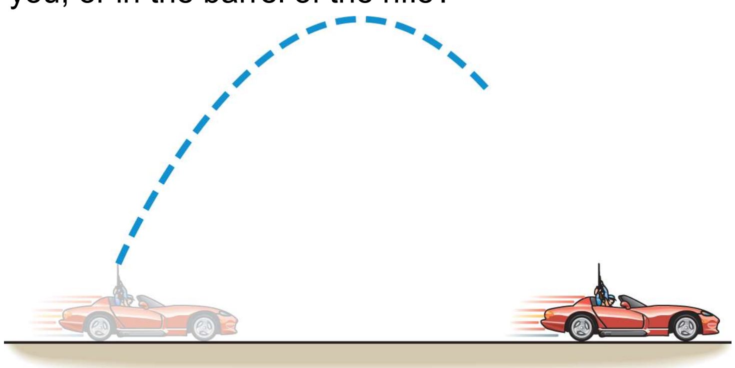
$$= -143\,\mathrm{m/s}$$

$$v = \sqrt{v_x^2 + v_y^2}$$

3.3 Projectile Motion

Conceptual Example 5 I Shot a Bullet into the Air...

Suppose you are driving a convertible with the top down. The car is moving to the right at constant velocity. You point a rifle straight up into the air and fire it. In the absence of air resistance, where would the bullet land – behind you, ahead of you, or in the barrel of the rifle?



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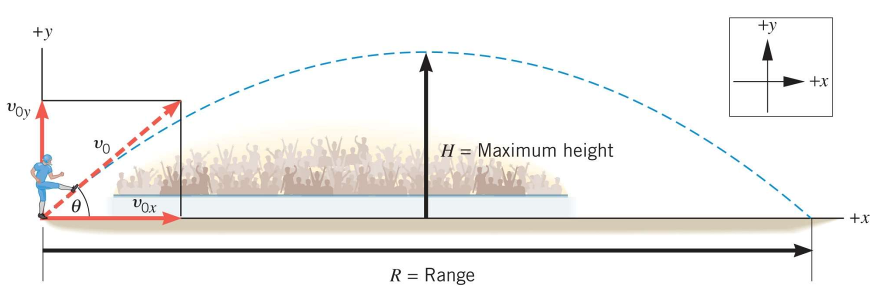
3.3 Projectile Motion

Example 6 The Height of a Kickoff

A placekicker kicks a football at and angle of 40.0 degrees and the initial speed of the ball is 22 m/s. Ignoring air resistance, determine the maximum height that the ball attains.



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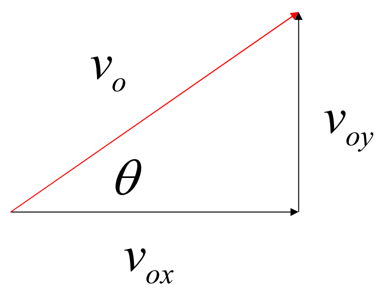
3.3 *Projectile Motion*



$$v_{oy} = v_o \sin\theta = (22\,\text{m/s})\sin 40^\circ = 14\,\text{m/s}$$

$$v_{ox} = v_o \sin\theta = (22\,\text{m/s})\cos 40^\circ = 17\,\text{m/s}$$

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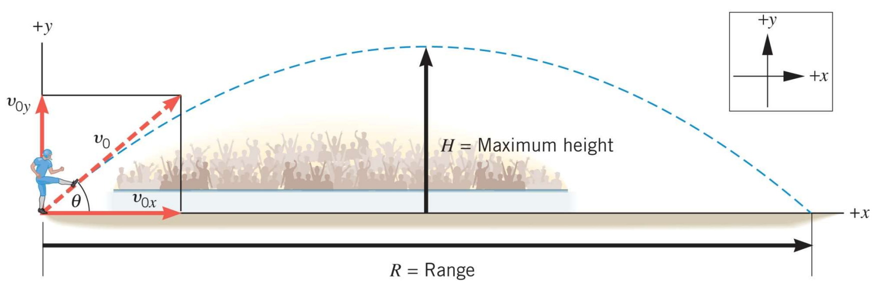
3.3 *Projectile Motion*



y	a_y	v_y	v_{oy}	t
?	-9.80 m/s^2	0	14 m/s	

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3.3 **Projectile Motion**

y	a_y	v_y	v_{oy}	t
?	-9.80 m/s^2	0	14 m/s	

$$v_y^2 = v_{oy}^2 + 2a_y y \quad \Longrightarrow \quad y = \frac{v_y^2 - v_{oy}^2}{2a_y}$$

$$y = \frac{0 - (14\,\text{m/s})^2}{2(-9.8\,\text{m/s}^2)} = +10\,\text{m}$$

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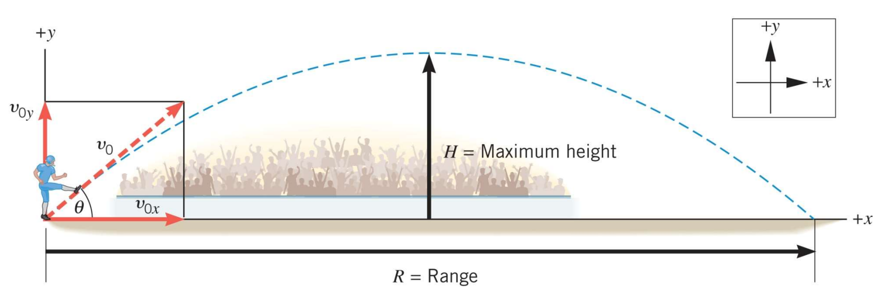
3.3 **Projectile Motion**

Example 7 The Time of Flight of a Kickoff

What is the time of flight between kickoff and landing?



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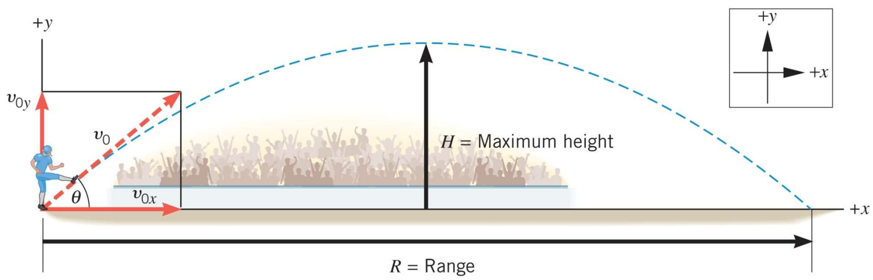
3.3 Projectile Motion



y	a_y	v_y	v_{oy}	t
0	-9.80 m/s^2		14 m/s	?

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3.3 Projectile Motion

y	a_y	v_y	v_{oy}	t
0	-9.80 m/s^2		14 m/s	?

$$y = v_{oy}t + \tfrac{1}{2}a_y t^2$$

$$0 = (14\,\text{m/s})t + \tfrac{1}{2}(-9.80\,\text{m/s}^2)t^2$$

$$0 = 2(14\,\text{m/s}) + (-9.80\,\text{m/s}^2)t$$

$$t = 0, \quad t = 2.9\,\text{s}$$

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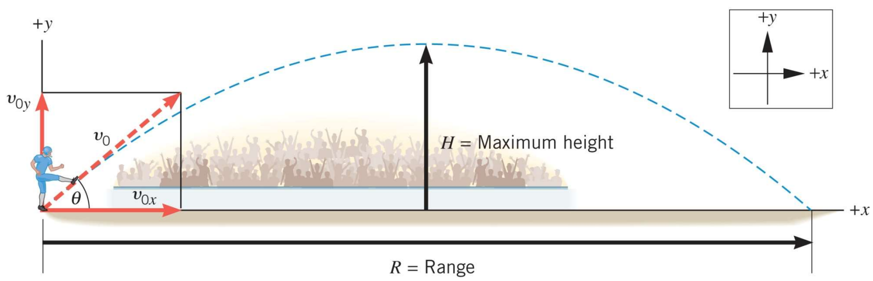
3.3 ***Projectile Motion***

Example 8 The Range of a Kickoff

Calculate the range R of the projectile.



$$x = v_{ox}t + \tfrac{1}{2}a_x t^2 = v_{ox}t$$

$$= (17\,\text{m/s})(2.9\,\text{s}) = +49\,\text{m}$$

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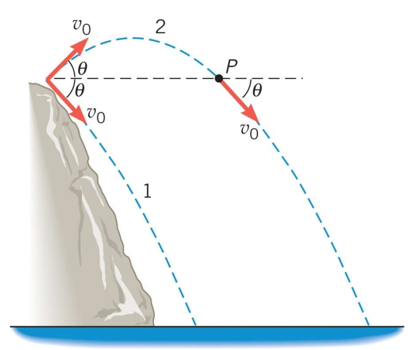
3.3 ***Projectile Motion***

Conceptual Example 10 Two Ways to Throw a Stone

From the top of a cliff, a person throws two stones. The stones have identical initial speeds, but stone 1 is thrown downward at some angle above the horizontal and stone 2 is thrown at the same angle below the horizontal. Neglecting air resistance, which stone, if either, strikes the water with greater velocity?



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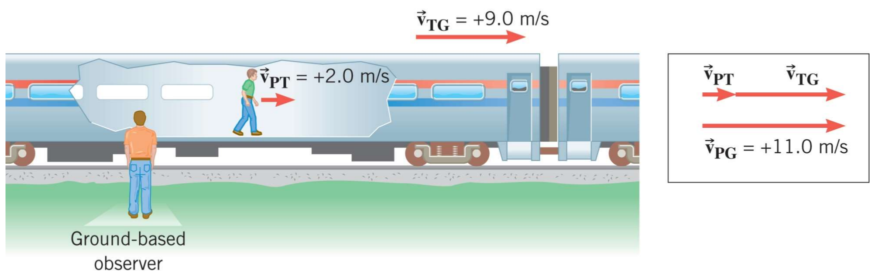
3.4 *Relative Velocity*



$$\vec{\mathbf{v}}_{PG} = \vec{\mathbf{v}}_{PT} + \vec{\mathbf{v}}_{TG}$$

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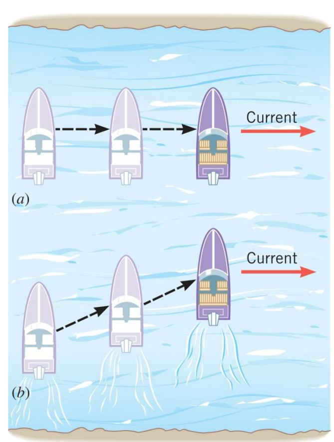
3.4 *Relative Velocity*

Example 11 Crossing a River

The engine of a boat drives it across a river that is 1800m wide. The velocity of the boat relative to the water is 4.0m/s directed perpendicular to the current. The velocity of the water relative to the shore is 2.0m/s.

(a) What is the velocity of the boat relative to the shore?

(b) How long does it take for the boat to cross the river?



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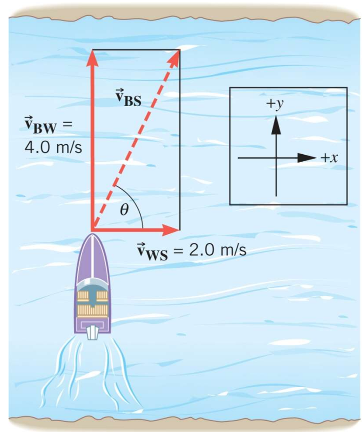
3.4 Relative Velocity

$$\vec{\mathbf{v}}_{\mathbf{BS}} = \vec{\mathbf{v}}_{\mathbf{BW}} + \vec{\mathbf{v}}_{\mathbf{WS}}$$

$$\theta = \tan^{-1}\left(\frac{4.0}{2.0}\right) = 63^{\circ}$$



$$v_{BS} = \sqrt{v_{BW}^2 + v_{WS}^2} = \sqrt{(4.0\,\text{m/s})^2 + (2.0\,\text{m/s})^2}$$
$$= 4.5\,\text{m/s}$$

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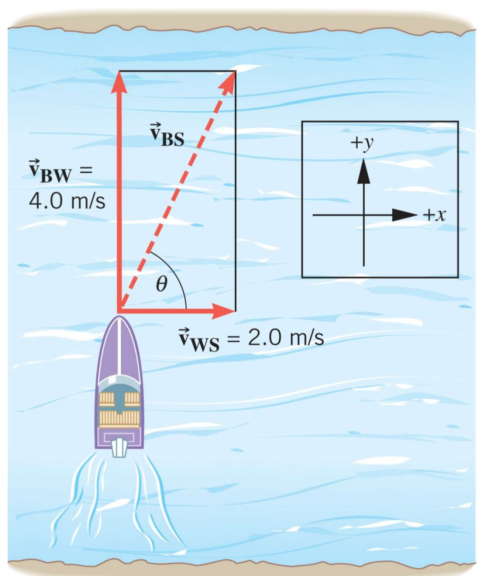
3.4 Relative Velocity

$$t = \frac{1800\,\text{m}}{4.0\,\text{m/s}} = 450\,\text{s}$$



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