Chapter 10

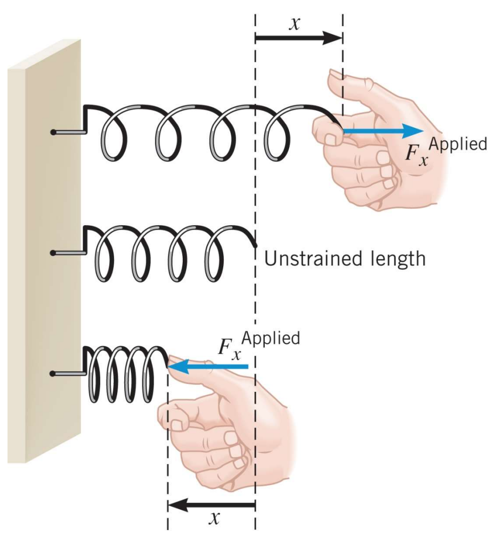
Simple Harmonic Motion and Elasticity

10.1 The Ideal Spring and Simple Harmonic Motion

$$F_x^{Applied} = k\,x$$

spring constant

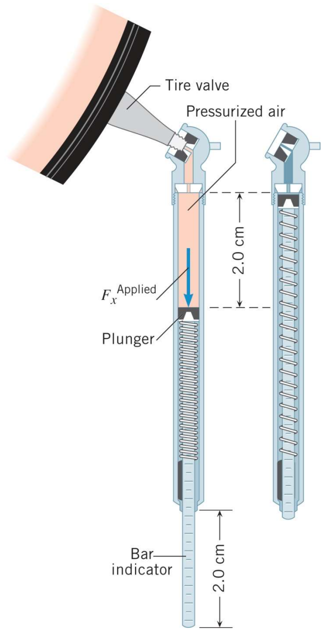
Units: N/m



10.1 *The Ideal Spring and Simple Harmonic Motion*

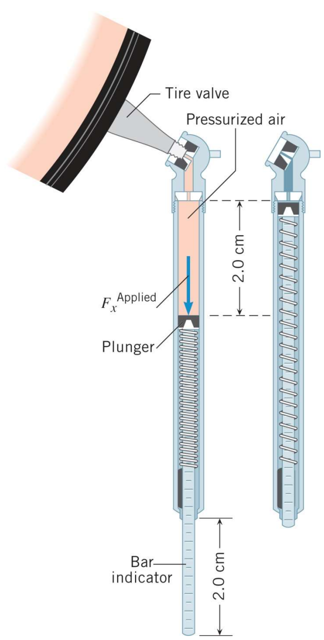
Example 1 A Tire Pressure Gauge

The spring constant of the spring is 320 N/m and the bar indicator extends 2.0 cm. What force does the air in the tire apply to the spring?



10.1 *The Ideal Spring and Simple Harmonic Motion*

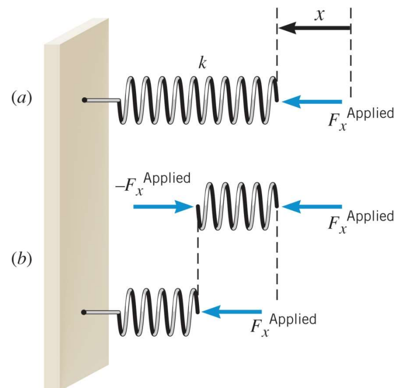
$$F_x^{Applied} = kx$$
$$= (320\,\text{N/m})(0.020\,\text{m}) = 6.4\,\text{N}$$



10.1 The Ideal Spring and Simple Harmonic Motion

Conceptual Example 2 Are Shorter Springs Stiffer?

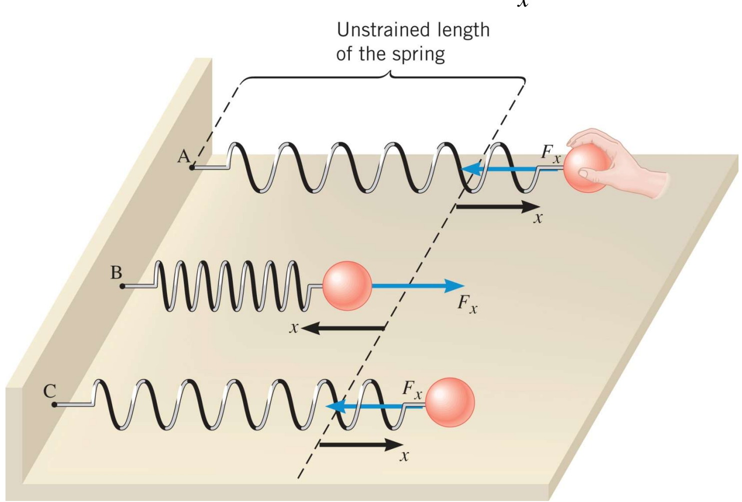
A 10-coil spring has a spring constant *k*. If the spring is cut in half, so there are two 5-coil springs, what is the spring constant of each of the smaller springs?



10.1 The Ideal Spring and Simple Harmonic Motion

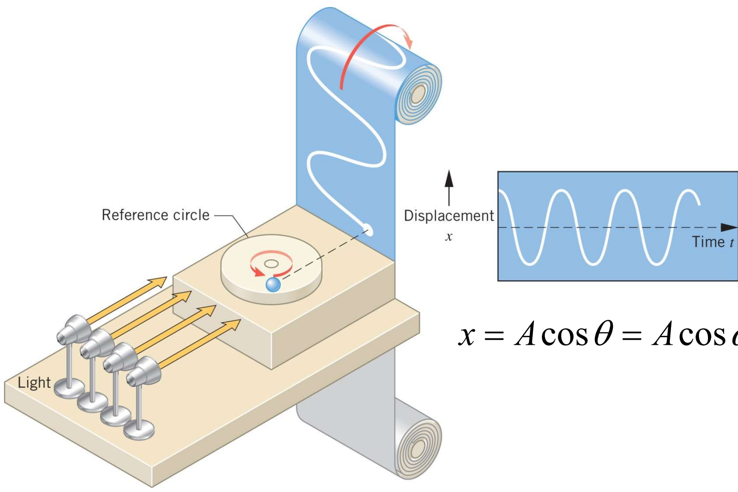
HOOKE'S LAW: RESTORING FORCE OF AN IDEAL SPRING

The restoring force on an ideal spring is $F_x = -kx$



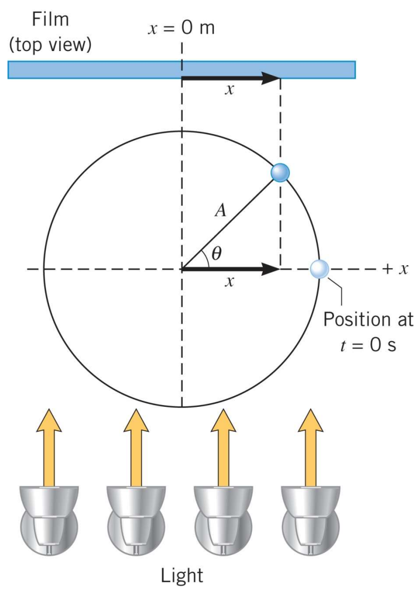
10.2 Simple Harmonic Motion and the Reference Circle

DISPLACEMENT



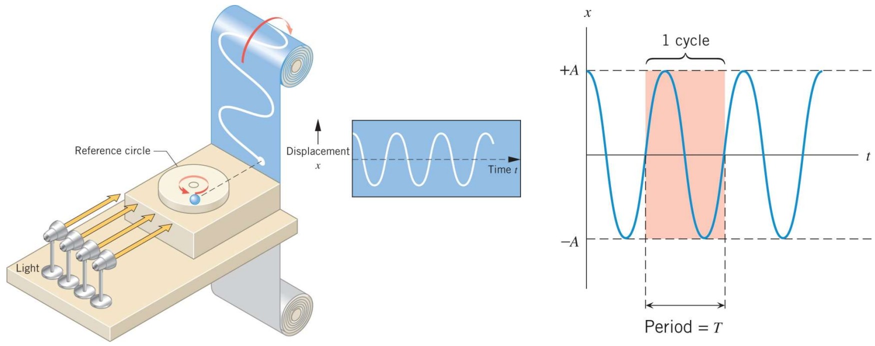
$$x = A\cos\theta = A\cos\omega t$$

10.2 Simple Harmonic Motion and the Reference Circle



$$x = A\cos\theta = A\cos\omega t$$

10.2 *Simple Harmonic Motion and the Reference Circle*



amplitude A: the maximum displacement

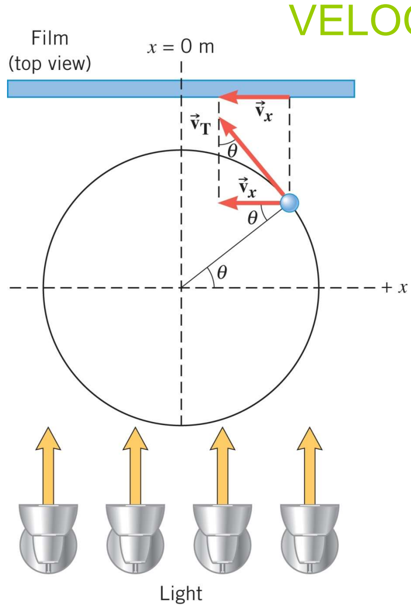
period T: the time required to complete one cycle

frequency f: the number of cycles per second (measured in Hz)

$$f = \frac{1}{T} \qquad \omega = 2\pi f = \frac{2\pi}{T}$$

10.2 *Simple Harmonic Motion and the Reference Circle*

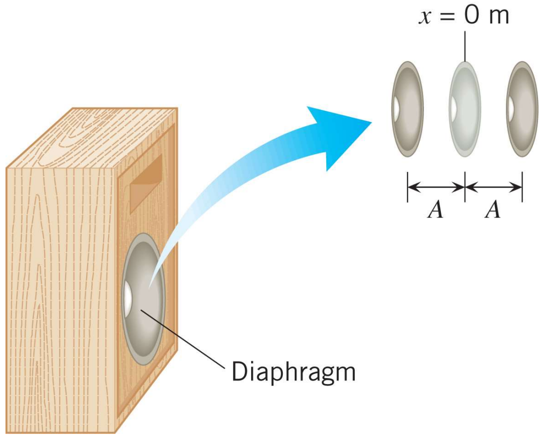
VELOCITY



$$v_x = -v_T \sin\theta = -\underbrace{A\omega}_{v_{max}} \sin\omega t$$

Example 3 The Maximum Speed of a Loudspeaker Diaphragm

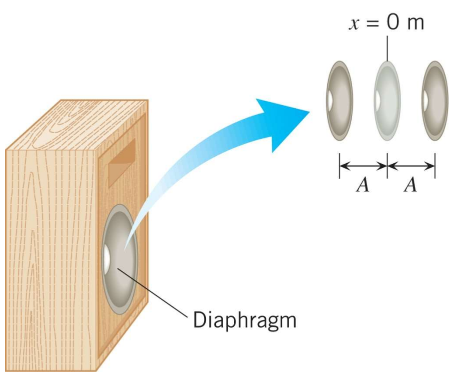
The frequency of motion is 1.0 KHz and the amplitude is 0.20 mm.
(a) What is the maximum speed of the diaphragm?
(b) Where in the motion does this maximum speed occur?



10.2 Simple Harmonic Motion and the Reference Circle

$$v_x = -v_T \sin\theta = -\underbrace{A\omega}_{v_{max}} \sin\omega t$$

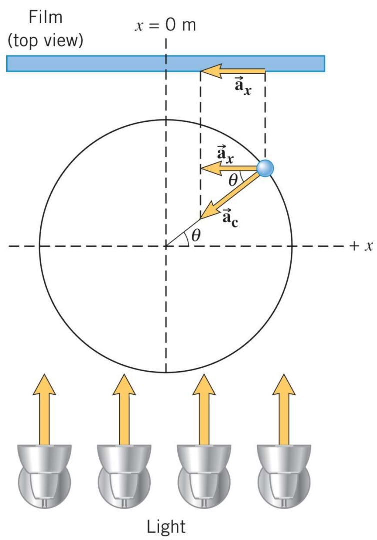
(a) $v_{max} = A\omega = A(2\pi f) = (0.20\times10^{-3}\,\text{m})(2\pi)(1.0\times10^{3}\,\text{Hz})$
$= 1.3\,\text{m/s}$



(b) The maximum speed occurs midway between the ends of its motion.

10.2 *Simple Harmonic Motion and the Reference Circle*

ACCELERATION



$$a_x = -a_c \cos\theta = -\underbrace{A\omega^2}_{a_{\max}} \cos\omega t$$

WILEY

10.2 *Simple Harmonic Motion and the Reference Circle*

FREQUENCY OF VIBRATION

$$x = A\cos\omega t \qquad a_x = -A\omega^2 \cos\omega t$$

$$\sum F = -kx = ma_x$$

$$-kA = -mA\omega^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

WILEY

10.2 Simple Harmonic Motion and the Reference Circle

Example 6 **A Body Mass Measurement Device**

The device consists of a spring-mounted chair in which the astronaut sits. The spring has a spring constant of 606 N/m and the mass of the chair is 12.0 kg. The measured period is 2.41 s. Find the mass of the astronaut.

Courtesy NASA

10.2 Simple Harmonic Motion and the Reference Circle

$$\omega = \sqrt{\frac{k}{m_{\text{total}}}} \quad \longrightarrow \quad m_{\text{total}} = k/\omega^2$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

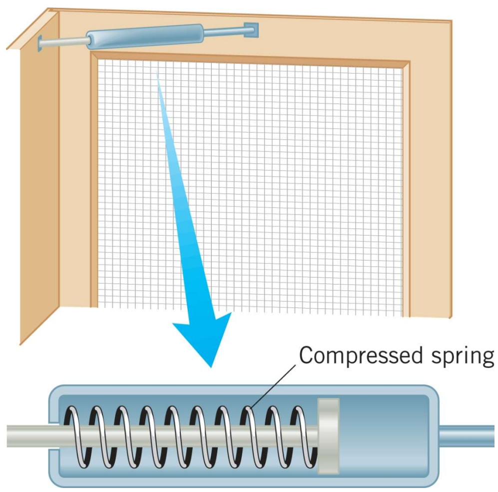
$$m_{\text{total}} = \frac{k}{(2\pi/T)^2} = m_{\text{chair}} + m_{\text{astro}}$$

$$m_{\text{astro}} = \frac{k}{(2\pi/T)^2} - m_{\text{chair}}$$

$$= \frac{(606\,\text{N/m})(2.41\,\text{s})^2}{4\pi^2} - 12.0\,\text{kg} = 77.2\,\text{kg}$$

10.3 Energy and Simple Harmonic Motion

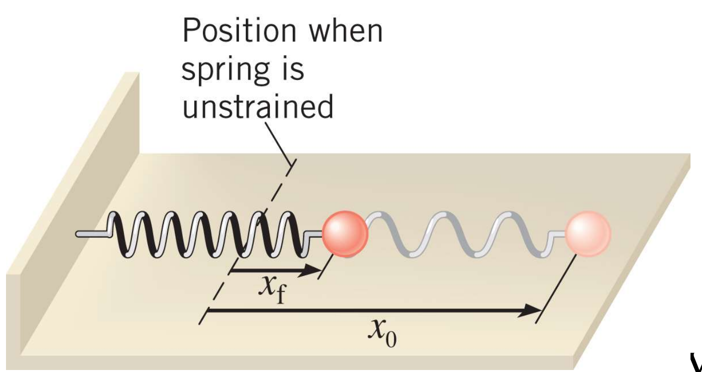
A compressed spring can do work.



10.3 Energy and Simple Harmonic Motion

$$W_{\text{elastic}} = \left(\overline{F}\cos\theta\right)s = \tfrac{1}{2}\left(kx_o + kx_f\right)\cos 0^\circ\left(x_o - x_f\right)$$

$$W_{\text{elastic}} = \tfrac{1}{2}kx_o^2 - \tfrac{1}{2}kx_f^2$$



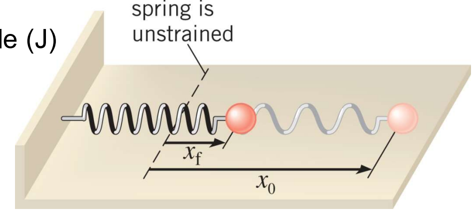
10.3 *Energy and Simple Harmonic Motion*

DEFINITION OF ELASTIC POTENTIAL ENERGY

The elastic potential energy is the energy that a spring has by virtue of being stretched or compressed. For an ideal spring, the elastic potential energy is

$$\mathrm{PE}_{\text{elastic}} = \tfrac{1}{2}kx^2$$

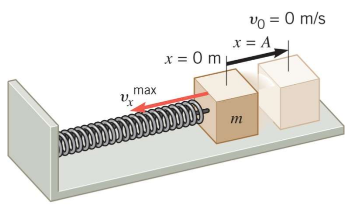
SI Unit of Elastic Potential Energy: joule (J)



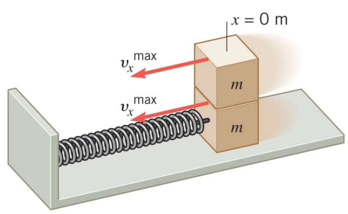
10.3 *Energy and Simple Harmonic Motion*

Conceptual Example 8 **Changing the Mass of a Simple Harmonic Oscillator**

The box rests on a horizontal, frictionless surface. The spring is stretched to x=A and released. When the box is passing through x=0, a second box of the same mass is attached to it. Discuss what happens to the (a) maximum speed (b) amplitude (c) angular frequency.



(a)

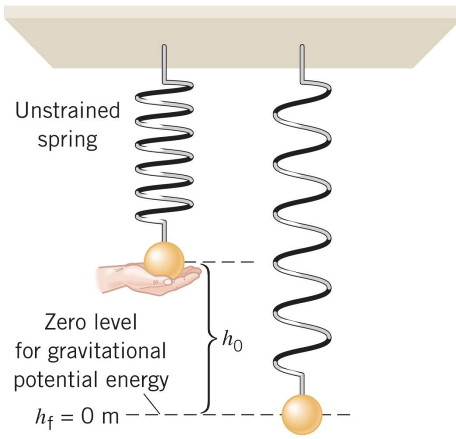


(b)

10.3 *Energy and Simple Harmonic Motion*

***Example 8* Adding a Mass to a Simple Harmonic Oscillator**

A 0.20-kg ball is attached to a vertical spring. The spring constant is 28 N/m. When released from rest, how far does the ball fall before being brought to a momentary stop by the spring?



10.3 *Energy and Simple Harmonic Motion*

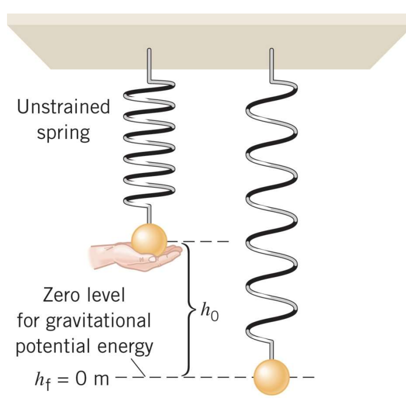
$$E_f = E_o$$

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgh_f + \tfrac{1}{2}ky_f^2 = \tfrac{1}{2}mv_o^2 + \tfrac{1}{2}I\omega_o^2 + mgh_o + \tfrac{1}{2}ky_o^2$$

$$\tfrac{1}{2}kh_o^2 = mgh_o$$

$$h_o = \frac{2mg}{k}$$

$$= \frac{2(0.20\,\text{kg})(9.8\,\text{m/s}^2)}{28\,\text{N/m}} = 0.14\,\text{m}$$



10.4 The Pendulum

A ***simple pendulum*** consists of a particle attached to a frictionless pivot by a cable of negligible mass.

$$\omega = \sqrt{\frac{g}{L}} \quad \text{(small angles only)}$$

Physical pendulum

$$\omega = \sqrt{\frac{mgL}{I}} \quad \text{(small angles only)}$$

WILEY

10.4 The Pendulum

Example 10 **Keeping Time**

Determine the length of a simple pendulum that will swing back and forth in simple harmonic motion with a period of 1.00 s.

$$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{g}{L}} \quad \longrightarrow \quad L = \frac{T^2 g}{4\pi^2}$$

$$L = \frac{T^2 g}{4\pi^2} = \frac{(1.00\,\text{s})^2 (9.80\,\text{m/s}^2)}{4\pi^2} = 0.248\,\text{m}$$

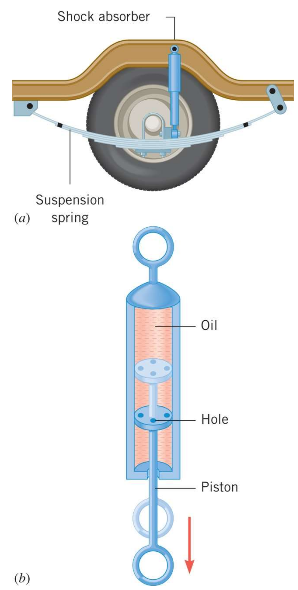
WILEY

10.5 Damped Harmonic Motion

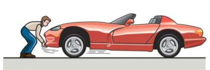
In simple harmonic motion, an object oscillates with a constant amplitude.

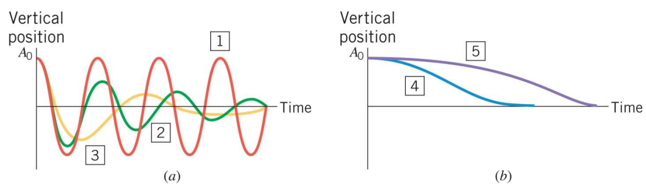
In reality, friction or some other energy dissipating mechanism is always present and the amplitude decreases as time passes.

This is referred to as ***damped harmonic motion***.



10.5 Damped Harmonic Motion





1) simple harmonic motion

2&3) underdamped

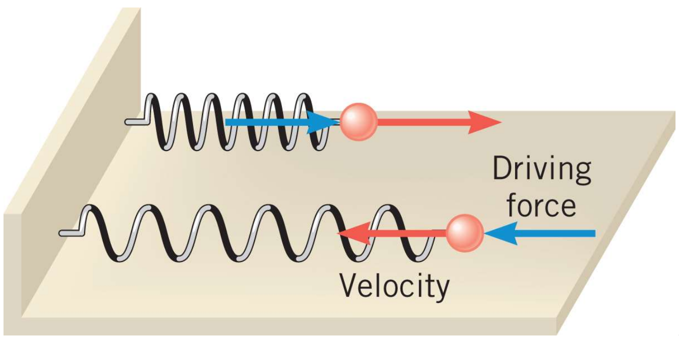
4) critically damped

5) overdamped

10.6 *Driven Harmonic Motion and Resonance*

When a force is applied to an oscillating system at all times, the result is ***driven harmonic motion***.

Here, the driving force has the same frequency as the spring system and always points in the direction of the object's velocity.

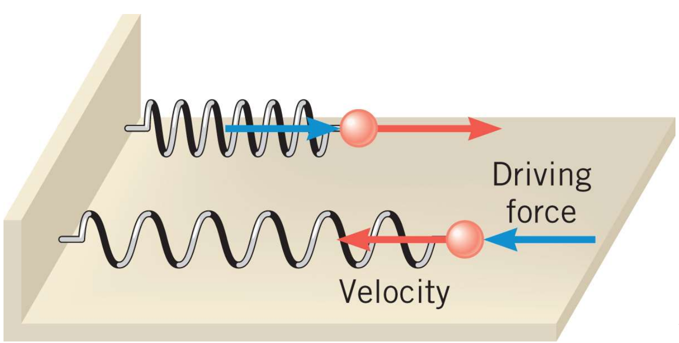


10.6 *Driven Harmonic Motion and Resonance*

RESONANCE

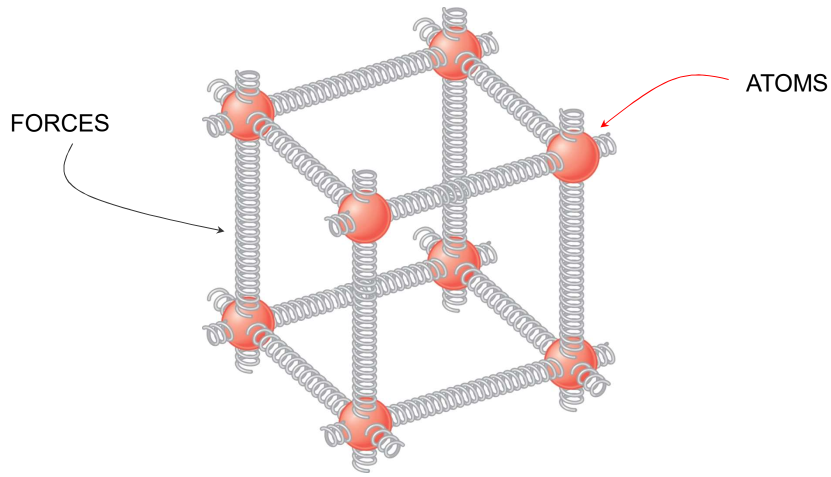
Resonance is the condition in which a time-dependent applied force can transmit large amounts of energy to an oscillating object, leading to a large amplitude motion.

Resonance occurs when the frequency of the applied force matches a natural frequency at which the object will oscillate.



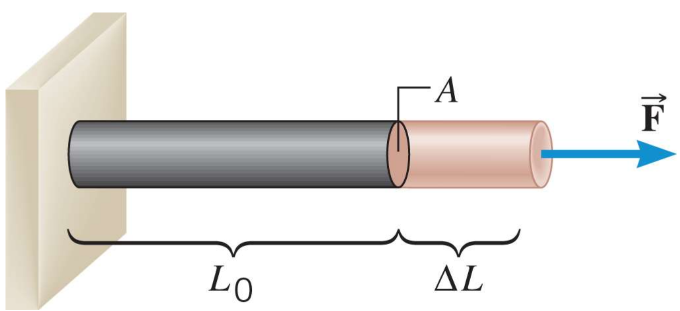
10.7 *Elastic Deformation*

Because of these atomic-level "springs", a material tends to return to its initial shape once forces have been removed.



10.7 *Elastic Deformation*

STRETCHING, COMPRESSION, AND YOUNG'S MODULUS



$$F = Y\left(\frac{\Delta L}{L_o}\right)A$$

Young's modulus has the units of pressure: N/m^2

Table 10.1 Values for the Young's Modulus of Solid Materials

Material	Young's Modulus Y (N/m^2)
Aluminum	6.9×10^{10}
Bone	
Compression	9.4×10^{9}
Tension	1.6×10^{10}
Brass	9.0×10^{10}
Brick	1.4×10^{10}
Copper	1.1×10^{11}
Mohair	2.9×10^{9}
Nylon	3.7×10^{9}
Pyrex glass	6.2×10^{10}
Steel	2.0×10^{11}
Teflon	3.7×10^{8}
Titanium	1.2×10^{11}
Tungsten	3.6×10^{11}

WILEY

Example 12 Bone Compression

In a circus act, a performer supports the combined weight (1080 N) of a number of colleagues. Each thighbone of this performer has a length of 0.55 m and an effective cross sectional area of 7.7×10^{-4} m^2. Determine the amount that each thighbone compresses under the extra weight.

Uwe Lein/©AP/Wide World Photos

WILEY

10.7 Elastic Deformation

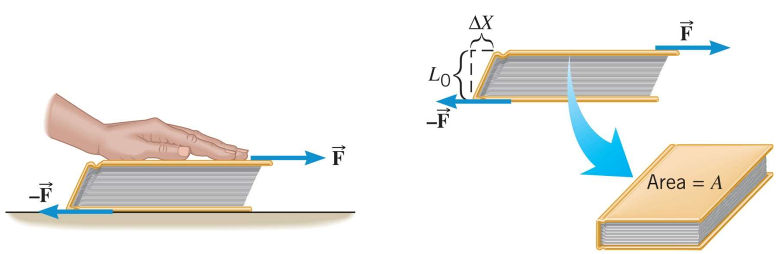
$$F = Y\left(\frac{\Delta L}{L_o}\right)A$$

$$\Delta L = \frac{FL_o}{YA} = \frac{(540\ \text{N})(0.55\ \text{m})}{(9.4\times10^{9}\ \text{N/m}^2)(7.7\times10^{-4}\ \text{m}^2)} = 4.1\times10^{-5}\ \text{m}$$

WILEY

10.7 Elastic Deformation

SHEAR DEFORMATION AND THE SHEAR MODULUS



(a) (b)

$$F = S\left(\frac{\Delta x}{L_o}\right)A$$

The shear modulus has the units of pressure: N/m^2

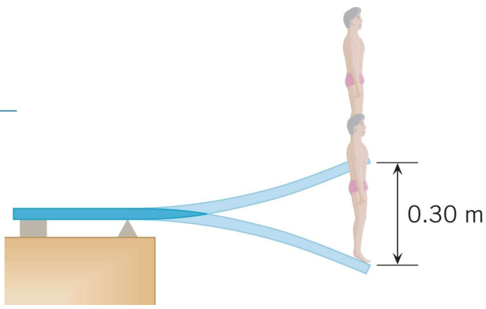
WILEY

Copyright © 2015 John Wiley & Sons, Inc. All rights reserved.

10.7 ***Elastic Deformation***

Table 10.2 Values for the Shear Modulus of Solid Materials

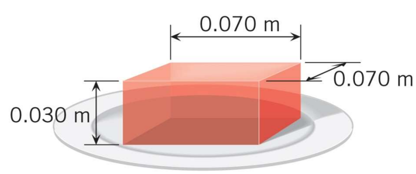
Material	Shear Modulus S (N/m^2)
Aluminum	2.4×10^{10}
Bone	1.2×10^{10}
Brass	3.5×10^{10}
Copper	4.2×10^{10}
Lead	5.4×10^{9}
Nickel	7.3×10^{10}
Steel	8.1×10^{10}
Tungsten	1.5×10^{11}



10.7 ***Elastic Deformation***

Example 14 **J-E-L-L-O**

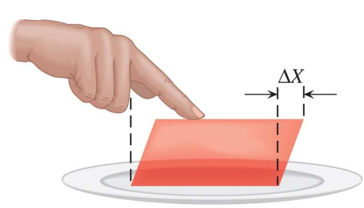
You push tangentially across the top surface with a force of 0.45 N. The top surface moves a distance of 6.0 mm relative to the bottom surface. What is the shear modulus of Jell-O?



(*a*)

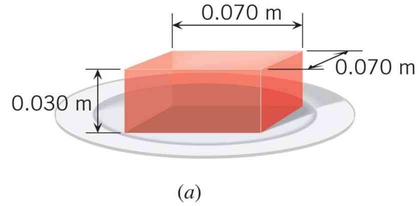
$$F = S\left(\frac{\Delta x}{L_o}\right)A$$

$$S = \frac{FL_o}{A\Delta x}$$

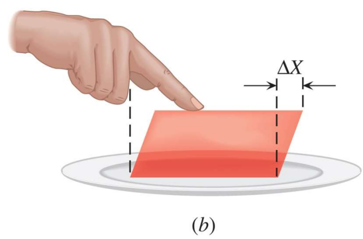


(*b*)

10.7 Elastic Deformation



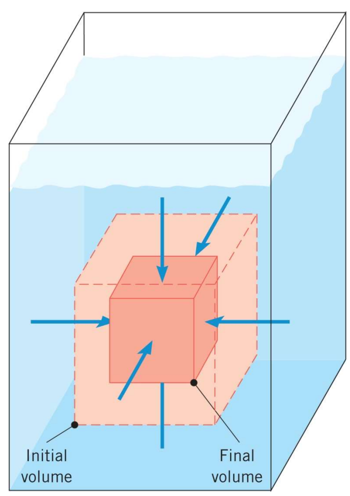
$$S = \frac{FL_o}{A\Delta x}$$



$$S = \frac{(0.45\text{ N})(0.030\text{ m})}{(0.070\text{ m})^2(6.0\times10^{-3}\text{ m})} = 460\text{ N/m}^2$$

10.7 Elastic Deformation

VOLUME DEFORMATION AND THE BULK MODULUS



$$\Delta P = -B\left(\frac{\Delta V}{V_o}\right)$$

The Bulk modulus has the units of pressure: N/m^2

10.7 *Elastic Deformation*

Table 10.3 Values for the Bulk Modulus of Solid and Liquid Materials

Material	Bulk Modulus *B* [N/m^2 (=Pa)]
Solids	
Aluminum	7.1×10^{10}
Brass	6.7×10^{10}
Copper	1.3×10^{11}
Diamond	4.43×10^{11}
Lead	4.2×10^{10}
Nylon	6.1×10^{9}
Osmium	4.62×10^{11}
Pyrex glass	2.6×10^{10}
Steel	1.4×10^{11}
Liquids	
Ethanol	8.9×10^{8}
Oil	1.7×10^{9}
Water	2.2×10^{9}

WILEY

10.8 *Stress, Strain, and Hooke's Law*

In general the quantity *F/A* is called the **stress**.

The change in the quantity divided by that quantity is called the strain:

$$\Delta V/V_o \quad \Delta L/L_o \quad \Delta x/L_o$$

HOOKE'S LAW FOR STRESS AND STRAIN

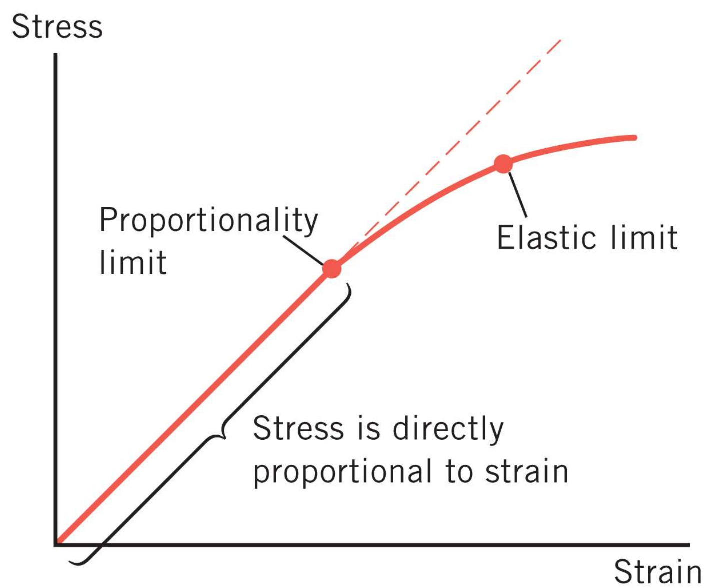
Stress is directly proportional to strain.

Strain is a unitless quantity.

SI Unit of Stress: N/m^2

WILEY

10.8 Stress, Strain, and Hooke's Law



WILEY