Update your skills. See page 404 III, IV.

1-1 Rational and Irrational Numbers

Objective To identify types of rational numbers • To find square roots of perfect squares • To approximate square roots of nonperfect squares • To recognize irrational numbers

During a 4-month period, an environmentalist recorded the following changes to a lake's water level: 4 in., $-2\frac{1}{4}$ in., 0.41 in., and $\frac{3}{8}$ in.

The numbers recorded above are *rational numbers*.

► A rational number is the quotient of two integers, a and b, written $\frac{a}{b}$, with $b \neq 0$.

All of the following types of numbers are rational numbers.

- *Integers*, which are whole numbers and their opposites, are represented by the set $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.
- *Fractions* and *mixed numbers*, such as $\frac{3}{8}$, $-\frac{11}{7}$, and $2\frac{1}{4}$, can be positive or negative.
- *Decimals* can also be positive or negative. They are either *terminating* or *repeating*.

  Terminating decimals, such as 0.41, 3.0, −5.7, have a finite number of digits.

  Repeating decimals, like the examples that follow, have a sequence of one or more digits that repeat indefinitely.

  2.333 . . . , which can be written as $2.\overline{3}$, and −0.636363 . . . , or $-0.\overline{63}$

> **Remember:**
> Natural, or Counting, Numbers: {1, 2, 3, 4, . . .}
> Whole Numbers: {0, 1, 2, 3, 4, . . .}
> Every integer a can be written as $\frac{a}{1}$.

Use an *ellipsis* (three dots) or use an overbar to show that one or more digits repeat in a decimal.

► Multiplying a number by itself, or raising it to the second power, is finding the *square* of that number. Some examples are given below.

square of $\frac{3}{8}$ → $\left(\frac{3}{8}\right)^2 = \frac{3}{8} \bullet \frac{3}{8} = \frac{9}{64}$

square of −7 → $(-7)^2 = -7 \bullet -7 = 49$

square of 0.5 → $(0.5)^2 = 0.5 \bullet 0.5 = 0.25$

opposite of the square of 4 → $-4^2 = -(4 \bullet 4) = -16$

Perfect squares, or square numbers, are the squares of natural numbers. Some examples of perfect squares are shown in the table below.

Natural Number	1	2	3	4	5	6	7	8	9	10
Square of the Number	1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2
Perfect Square	1	4	9	16	25	36	49	64	81	100

The set of perfect squares is {1, 4, 9, 16, 25, 36, 49, 64, 81, 100, . . .}.

► A square root of a number is one of two equal factors of that number. The positive square root of a number is called the principal square root. It is indicated with the symbol $\sqrt{\ }$, called a radical sign. The expression under a radical sign is called the radicand. The negative square root of a number is indicated by writing a negative sign in front of the radical.

$\sqrt{25} = \sqrt{5 \bullet 5} = 5$ ← principal square root

Read as "the principal square root of 25."

$-\sqrt{25} = -5$ ← negative square root

Read as "the negative square root of 25."

Examples

Find each square root.

1 $-\sqrt{225}$

$-\sqrt{225} = -\sqrt{3 \bullet 3 \bullet 5 \bullet 5}$

$= -\sqrt{(3 \bullet 5)(3 \bullet 5)}$

$= -(3 \bullet 5) = -15$

Remember: The prime factorization of a number shows the number as the product of prime factors.

2 $\sqrt{\frac{4}{9}}$

$\sqrt{\frac{4}{9}} = \sqrt{\left(\frac{2}{3}\right)^2}$

$= \sqrt{\frac{2}{3} \bullet \frac{2}{3}} = \frac{2}{3}$

Think What number squared equals $\frac{4}{9}$?

▶ To *approximate* the square root of a nonperfect square, a number that is not the square of a natural number, find two consecutive integers that the square root is between.

Between what two consecutive integers is $\sqrt{19}$?

19 is between 16 and 25. ← Find two nearby perfect squares that 19 is between.

$\sqrt{19}$ is between $\sqrt{16}$ and $\sqrt{25}$.

$\sqrt{16} = 4$ and $\sqrt{25} = 5$ ← Find each square root.

So $\sqrt{19}$ is between 4 and 5.

Think The perfect squares in order are 1, 4, 9, 16, 25, ...
19 is between 16 and 25.

▶ Square roots of nonperfect squares are examples of numbers that are not rational. Numbers that are not rational are called *irrational*. The following are irrational numbers.

- positive or negative decimals that are *nonterminating* and *nonrepeating* or have a pattern in their digits but do not repeat exactly.
  4.13216582 . . . , −0.5050050005 . . .
- square roots of nonperfect squares
  $\sqrt{42}, -\sqrt{250}$
- pi, symbolized by the Greek letter π (3.14 and $\frac{22}{7}$ are rational *approximate* values.)

Key Concept

Irrational Numbers

Irrational numbers are numbers that *cannot* be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Try These

For each number, list all the terms that apply: *whole number*, *integer*, *rational number*, and *irrational number*.

1. −321.11 **2.** 45 **3.** 0.010203 . . . **4.** $1.\overline{358}$

5. $\frac{22}{7}$ **6.** $\sqrt{144}$ **7.** $-\sqrt{36}$ **8.** $\sqrt{102}$

Find each square root. If the radicand is a nonperfect square, between which two consecutive integers would the square root fall?

9. $\sqrt{400}$ **10.** $\sqrt{205}$ **11.** $-\sqrt{\frac{49}{81}}$ **12.** $-\sqrt{77}$

13. A contractor is building a patio in the shape of a square. The patio will cover 945 square feet. Estimate the length of the side of the patio to the nearest integer.

14. **Discuss and Write** Which of these numbers is irrational: $\sqrt{36}, \sqrt{\frac{1}{36}}, \sqrt{3.6}$? Explain.

Go to PRACTICE BOOK Lesson 1-1 for exercise sets.