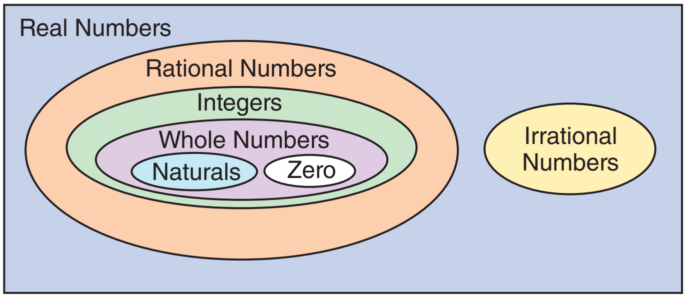
1-2

Update your skills. See page 406, VII.

The Set of Real Numbers

Objective **To classify real numbers • To graph real numbers on a number line • To compare and order real numbers • To find the absolute value and additive inverse of a real number • To understand and apply the Closure Property**

The set of real numbers consists of all rational numbers and irrational numbers. The Venn diagram at the right shows the relationships among natural numbers, whole numbers, integers, rational numbers, and irrational numbers.



To which sets of numbers does each one of these real numbers belong: $0.5\overline{4}$, $-\sqrt{81}$, $\frac{44}{11}$, and $\sqrt{98}$?

▶ To decide to which sets of numbers a real number belongs, you may need to rename the number in a different form.

- $0.5\overline{4} = 0.5444\ldots$ ← a repeating decimal
  rational number
- $\frac{44}{11} = 4$ ← a natural number
  natural number; whole number; integer; rational number
- $-\sqrt{81} = -9$ ← an integer
  integer; rational number
- $\sqrt{6}$ ← a nonperfect square radicand
  irrational number

▶ There is a *one-to-one correspondence* between the set of real numbers and the points on a number line. This is illustrated by the Completeness Property for Points on the Number Line.

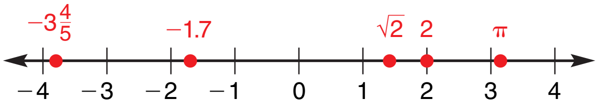
Key Concept

Completeness Property for Points on the Number Line

Every real number corresponds to exactly one point on a number line, and every point on the number line corresponds to exactly one real number.

The *origin* of a number line is zero. Points to the right of zero correspond to *positive numbers.* Points to the left of zero correspond to *negative numbers.* Positive and negative numbers are often called *signed numbers.* The number zero is neither positive nor negative.

The point that corresponds to a real number is called the *graph of the number.* The number line below shows the graph of $\sqrt{2}$, $-3\frac{4}{5}$, π, 2, and -1.7.



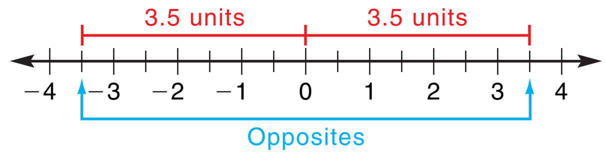
▶ A number line can help you compare and order real numbers. The farther to the right a number is on the number line, the greater it is.

Use the number line above to compare and order the following numbers from least to greatest.

$-1.7 > -3\frac{4}{5}$ $\qquad$ $\sqrt{2} < \pi$ $\qquad$ 2 *is between* $\sqrt{2}$ and π.

In order from least to greatest: $-3\frac{4}{5}, -1.7, \sqrt{2}, 2, \pi$

▶ The absolute value of a real number, n, written $|n|$, is the distance from 0 to n on a number line. Since distance is always positive, the absolute value of a nonzero number is always positive.



The absolute value of 3.5 is 3.5. **Write:** $|3.5| = 3.5$

The absolute value of −3.5 is 3.5. **Write:** $|-3.5| = 3.5$

▶ Two real numbers are *opposites* (or additive inverses) if they are on opposite sides of 0 and they are the same distance from 0 on a number line. The sum of a real number and its additive inverse is 0.

Key Concept

Additive Inverse Property
For any real number a,
$a + (-a) = 0$ and $-a + a = 0$.

The opposite (or additive inverse) of 3.5 is −3.5. **Write:** $-(3.5) = -3.5$
$3.5 + (-3.5) = 0$

The opposite (or additive inverse) of −3.5 is 3.5. **Write:** $-(-3.5) = 3.5$
$-3.5 + [-(-3.5)] = 0$

The opposite (or additive inverse) of 0 is 0. **Write:** $-(0) = 0$
$0 + 0 = 0$

▶ You can find the value of an expression involving absolute values or additive inverses of real numbers.

Find the value of each expression.

$-(-30) \bullet -(-12)$
$30 \bullet 12$ ← Find the opposite of each factor.
360

$|-6.9| - |2.9|$
$6.9 - 2.9$ ← Find the absolute value of each number.
4

▶ When you apply an operation (for example, addition) to any numbers in a set and the result is also a number of that set, the set is said to be *closed* under the operation. This is called the Closure Property.

Finding a counterexample that shows that a set of numbers is *not* closed under a given operation is one way to test closure for that set under the given operation. A single counterexample proves that a statement is false.

- The set of real numbers is closed under addition.
  True, whenever you add two real numbers, the sum is always a real number.
- The set of integers is closed under division.
  False, $3 \div 5 = \frac{3}{5}$ ← $\frac{3}{5}$ is *not* an integer.

Try These

Give an example to illustrate the type of number described.

1. a real number that is not rational
2. a rational number that is not an integer

Graph the numbers on a number line. Then write the numbers in order from least to greatest.

3. $-2.2, \sqrt{9}, 0, -\frac{5}{2}, -1.\overline{3}$

Find the value of the expression.

4. $-|9 \bullet 75|$
5. $-|3.9 + 5.2|$

6. **Discuss and Write** Explain why $\{-1, 1\}$ is closed under division.

Go to PRACTICE BOOK Lesson 1-2 for exercise sets.