Name _____

1-5A Properties of Integer Exponents

Objective To summarize properties of exponents • To use properties of exponents to simplify expressions

The list at the right shows the Properties of Exponents that can be used to simplify expressions with integer exponents. Read each property carefully and focus on the **Power of a Power Property**, the **Power of a Product Property**, and the **Power of a Quotient Property**.

A company makes boxes shaped like cubes. One box has edges that are 5 inches long. A larger box has edges that are 3 times as long. What is the volume of the larger box?

▶ To find the volume of the box, first use the formula for the volume of a cube. Then use the Power of a Product Property.

$V = e^3$ ← Write the formula for the volume of a cube.

$= (3 \bullet 5)^3$ ← Substitute the edge length of the larger box.

$= 3^3 \bullet 5^3$ ← Use the Power of a Product Property.

$= 27 \bullet 125$ ← Evaluate the powers.

$= 3375$ ← Simplify the expression.

So, the volume of the larger box is 3375 square inches.

Properties of Exponents

For any real numbers a & b and integers m & n:

Definition of Zero Exponent
$a^0 = 1, a \neq 0$

Definition of Negative Exponent
$a^{-m} = \frac{1}{a^m}, a \neq 0$

Law of Exponents for Multiplication
$a^m \bullet a^n = a^{m+n}, a \neq 0$

Law of Exponents for Division
$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$

Power of a Power Property
$(a^m)^n = a^{m \bullet n}$

Power of a Product Property
$(ab)^m = a^m \bullet b^m$

Power of a Quotient Property
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

Examples

Simplify $\left(\frac{2}{3}\right)^4$.

$\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$ ← Use the Power of a Quotient Property.

$= \frac{2 \bullet 2 \bullet 2 \bullet 2}{3 \bullet 3 \bullet 3 \bullet 3}$ ← Write the powers as products.

$= \frac{16}{81}$ ← Simplify.

Simplify $(4^2)^{-3}$.

$(4^2)^{-3} = 4^{2 \bullet (-3)}$ ← Use the Power of a Power Property.

$= 4^{-6}$ ← Multiply.

$= \frac{1}{4^6}$ ← Apply the Definition of a Negative Exponent.

$= \frac{1}{4096}$ ← Use a handheld. Simplify.

Simplify. Tell which property or properties you used.

1. $(4 \bullet 3)^5$ _____

2. $(3^{-4})^2$ _____

3. $\left(\frac{3}{5}\right)^3$ _____

4. $(-2 \bullet 6)^{-2}$ _____

5. Discuss and Write Explain why the Power of a Quotient Property is not valid for cases when $b = 0$.

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1-5A Properties of Integer Exponents

Simplify. Express answers in standard form.

6. $\left(-\frac{1}{3}\right)^{-5}$ _____

7. $(-2 \bullet 3)^4$ _____

8. $(17^4)^0$ _____

9. $\left(\frac{5}{4}\right)^3$ _____

10. $(2 \bullet 2 \bullet 2)^5$ _____

11. $\left(10 \bullet \frac{5}{2}\right)^{-3}$ _____

12. $(-6 \bullet 3^2)^2$ _____

13. $((9^7)^3)^0$ _____

14. $\left(\frac{2}{5} \bullet \frac{7}{3}\right)^2$ _____

15. $\frac{(3^3 \bullet 6^2)^2}{(3 \bullet 6)^2}$ _____

16. $\left(\frac{3^{-2}}{4}\right)^{-3}$ _____

17. $\left(\frac{8^{-1}}{3^{-1}}\right)^4$ _____

Simplify.

18. $x^8x^{-5}x^2$ _____

19. $\left(\frac{k^2}{m}\right)^{-6}$ _____

20. $\left(\frac{n^{-3}}{n}\right)^4$ _____

21. $\frac{(r^2)^5}{s^2s^5}$ _____

22. $(-2y^2)^4$ _____

23. $\frac{3xy^4}{9x^{-1}y}$ _____

24. $\left(\frac{(a^{-2})^{-1}}{b^2}\right)^{-3}$ _____

25. $\frac{(c^4d)^3}{(cd^5)^{-1}}$ _____

26. $\frac{(p^3q^{-1})^2}{(p^{-2}q^3)^3}$ _____

Problem Solving

Solve. Show your work.

27. The side of a square measures $\frac{9}{10}$ cm. What is the area of the square? Express your answer as a fraction of a square centimeter, in lowest terms.

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28. The length, width, and height of a certain bacteria cell are all about 10^{-6} m. Use the formula for the volume of a cube to approximate the volume of the cell in cubic meters. Write your answer using a single exponent.

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29. Bob says that $a^2 \bullet a^2 \bullet a^2$ is less than or equal to $((a^2)^2)^2$ for all values of a. Find a counterexample to show Bob is incorrect.

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30. A beach ball filled with air has a diameter of $\frac{2}{3}$ ft. How many cubic feet of air are in the ball? Write your answer in terms of π. (Hint: Volume of a sphere = $\frac{4}{3}\pi r^3$.)

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31. Jolie reduces the size of a poster to $\frac{3}{4}$ its original size. She does this a total of 4 times. If the original poster was 2 feet wide, how wide is the final image?

_____

32. Randy is making a scale model of a sculpture shaped like a cube. The original cube has edges that are x m long. His model has edges that are $\frac{2}{5}$ that size. What is the volume of Randy's model?

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CRITICAL THINKING

33. Use the Law of Exponents for Multiplication to show that $(a^m)^n = a^{m \bullet n}$.

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