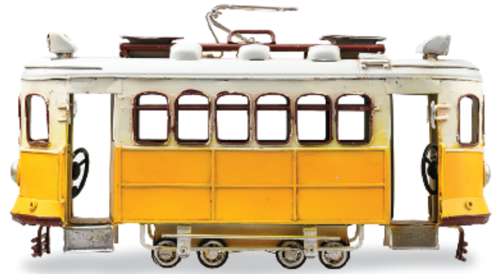
Update your skills. See page 403, I.

1-5 Integer Exponents

Objective To write repeated multiplication in exponential form and vice versa • To apply the Laws of Exponents for Multiplication and Division • To apply the definitions of zero and negative exponents

Mr. Toppler's miniature trolley model has 2 levels and 2 tracks per level. There are 2 trolleys per track, and each trolley has 2 cars. Each car has 2 pairs of wheels. How would you represent the total number of wheels the cars have using exponents?



To represent the total number of wheels using exponents, write the number as an expression involving repeated multiplication.

$$\underset{\text{number of levels}}{2} \bullet \underset{\text{number of tracks}}{2} \bullet \underset{\text{number of trolleys}}{2} \bullet \underset{\text{number of cars}}{2} \bullet \underbrace{2 \bullet 2}_{\text{number of wheels in one car}}$$

► You can write a repeated multiplication expression as an expression using exponents. An exponent tells how many times a number, called the base, is used as a factor. A number in exponent form, or its equivalent standard form, is a power of that number.

base exponent

$$2^6 = \underbrace{2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2}_{\text{2 is a factor 6 times}} = 64 \leftarrow \text{power of 2}$$

2^6: power of 2

Read 2^6 as:
"two to the sixth power" or
"the sixth power of two."

The cars have a total number of 2^6 wheels.

► The exponents 2 and 3 have special names.

$11^2 = 11 \bullet 11 = 121$

11^2: exponential form; 121: standard form

Read 11^2 as:
"11 squared,"
"11 to the second power," or "the second power of 11."

$(-3)^3 = -3 \bullet -3 \bullet -3 = -27$

$(-3)^3$: exponential form; -27: standard form

Read $(-3)^3$ as:
"negative three cubed,"
"negative three to the third power," or "the third power of negative three."

► To multiply powers with the *same base*, add the exponents.

Simplify using a single exponent: $(-5)^2(-5)^3$

$$(-5)^2(-5)^3 = \underbrace{(-5 \bullet -5)}_{\text{2 factors}}\underbrace{(-5 \bullet -5 \bullet -5)}_{\text{3 factors}} \leftarrow \text{List the factors.}$$

$$= \underbrace{(-5 \bullet -5 \bullet -5 \bullet -5 \bullet -5)}_{\text{(2 + 3) factors}} \leftarrow \text{Group the factors.}$$

$$= (-5)^{2+3} \leftarrow \text{Write in exponential form.}$$

$$= (-5)^5 \leftarrow \text{Simplify the exponent.}$$

Key Concept

Law of Exponents for Multiplication

For any real number a, $a \neq 0$, and integers m and n, $a^m \bullet a^n = a^{m+n}$.

▶ To divide powers with the *same base*, subtract the exponents.

Simplify using a single exponent: $9^5 \div 9^2$

$$\frac{9^5}{9^2} = \frac{\cancel{9}^{1} \bullet \cancel{9}^{1} \bullet 9 \bullet 9 \bullet 9}{\cancel{9}_{1} \bullet \cancel{9}_{1}} = \frac{9 \bullet 9 \bullet 9}{1}$$ ← List the factors and simplify.

$$= \frac{9^3}{1} = 9^3$$ ← Write in exponential form and simplify.

So $\frac{9^5}{9^2} = 9^{5-2} = 9^3$.

Key Concept

Law of Exponents for Division

For any real number a, $a \neq 0$, and integers m and n,

$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$.

▶ Study the pattern below. Note that the exponents of successive expressions decrease by 1 and the value of each exponential expression is one-third of the previous expression. The pattern shows that exponents can also be *zero* or *negative*.

$3^3 = 3 \bullet 3 \bullet 3$

$3^2 = 3 \bullet 3$

$3^1 = 3$

$3^0 = \frac{3}{3} = 1$

$3^{-1} = \frac{1}{3} = \frac{1}{3^1}$

$3^{-2} = \frac{1}{3 \bullet 3} = \frac{1}{3^2} = \frac{1}{9}$

$3^{-3} = \frac{1}{3 \bullet 3 \bullet 3} = \frac{1}{3^3} = \frac{1}{27}$

A zero exponent means the expression has a value of 1.

A negative exponent means the multiplicative inverse or reciprocal of the expression.

Key Concept

Zero and Negative Exponents

- For any nonzero number a, $a^0 = 1$.
- For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$.

Examples

Simplify.

1 $5 \bullet 5^{-4} \bullet 5^3$

Think: $5 = 5^1$

$5^1 \bullet 5^{-4} \bullet 5^3 = 5^{1+(-4)+3}$ ← Add the exponents with the same base.

$= 5^0$ ← Simplify exponents.

$= 1$ ← Apply the definition of zero exponent.

2 $\frac{5^4 \bullet 5^{-2}}{5^3}$

$\frac{5^4 \bullet 5^{-2}}{5^3} = \frac{5^{4+(-2)}}{5^3} = \frac{5^2}{5^3}$ ← Add the exponents with the same base.

$= 5^{2-3} = 5^{-1}$ ← Subtract the exponents with the same base.

$= \frac{1}{5}$ ← Apply the definition of negative exponent.

Try These

Simplify.

1. $6^2 \bullet 6 \bullet 6^{-4}$

2. $(-3)^5 \bullet (-3)^0 \bullet (-3)^{-6}$

3. $\frac{4 \bullet 4^0}{4^3 \bullet 4^{-1}}$

4. $\frac{\left(\frac{2}{3}\right)^3 \bullet \left(\frac{2}{3}\right)^{-7} \bullet \left(\frac{2}{3}\right)}{\left(\frac{2}{3}\right)^{-4} \bullet \left(\frac{2}{3}\right)^0}$

5. Discuss and Write Al correctly calculates that $2^4 = 4^2$, and assumes that $a^b = b^a$ is always true for $a, b \neq 0$. Is Al's assumption correct? Justify your answer.

Go to PRACTICE BOOK Lesson 1-5 for exercise sets.