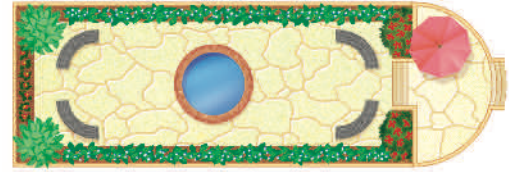


# The Order of Operations

**Objective** To simplify numerical expressions using the order of operations



At the far end of a park is a meditation garden. The width of the garden is 13.4 meters, and the length is 2.5 meters more than the width. If a new semicircular entrance to the garden will be added, as shown in the diagram at the right, what will be the total area of the new garden to the nearest hundredth?



To find the total area, write an expression to represent it, and then simplify by using the **order of operations**.

area of meditation garden

area of new entrance

**Think**

Area of semicircle:

$$A = \frac{1}{2}\pi r^2$$

$$13.4(13.4 + 2.5) + \frac{1}{2}(3.14)(6.7^2)$$

$$13.4(13.4 + 2.5) + \frac{1}{2}(3.14)(6.7^2) \leftarrow \text{Compute within parentheses first, then compute exponents.}$$

$$13.4(15.9) + \frac{1}{2}(3.14)(44.89) \leftarrow \text{Multiply next.}$$

$$213.06 + 70.4773 \leftarrow \text{Add last.}$$

$$283.5373 \leftarrow \text{Think } 283.5373 \approx 283.54$$

**Check:**

$$13.4 \times (13.4 + 2.5) + \frac{1}{2} \times 3.14 \times 6.7^2 \approx$$

So the total area of the new garden is 283.54 square meters.

► When an expression has several grouping symbols, simplify the expression within the *innermost grouping symbols* first.

$$\text{Simplify: } -2[-6 - 5(1.2 + 2^2 \cdot 3.5) - 8]$$

$$-2[-6 - 5(1.2 + 2^2 \cdot 3.5) - 8] = -2[-6 - 5(1.2 + 4 \cdot 3.5) - 8] \leftarrow \text{Compute exponent within parentheses.}$$

$$= -2[-6 - 5(1.2 + 14) - 8] \leftarrow \text{Next, multiply within parentheses.}$$

$$= -2[-6 - 5(15.2) - 8] \leftarrow \text{Then add within parentheses.}$$

$$= -2[-6 - 76 - 8] \leftarrow \text{Next, multiply within brackets.}$$

$$= -2[-90] \leftarrow \text{Then subtract within brackets.}$$

$$= 180 \leftarrow \text{Finally, multiply.}$$

innermost grouping symbols

- A fraction bar is also a grouping symbol. Do any computation above or below the fraction bar before simplifying the fraction.

Simplify:  $\frac{(11 - 2)^2}{3(2 + 1.5) - 1.5} - 3$

$$\frac{(11 - 2)^2}{3(2 + 1.5) - 1.5} - 3 = \frac{(9)^2}{3(3.5) - 1.5} - 3 \quad \leftarrow \text{Compute within parentheses first.}$$

$$= \frac{81}{10.5 - 1.5} - 3 \quad \leftarrow \text{Next, compute the exponent above, and multiply below the fraction bar.}$$

$$= \frac{81}{9} - 3 = 9 - 3 \quad \leftarrow \text{Subtract below the fraction bar. Divide next.}$$

$$= 6 \quad \leftarrow \text{Subtract last.}$$

## Examples

1 Simplify:  $\frac{3}{4}[2(5\frac{1}{2} - 3.5)]^3$

$$\frac{3}{4}[2(5\frac{1}{2} - 3.5)]^3 = \frac{3}{4}[2(2)]^3$$

$$= \frac{3}{4}[4]^3$$

$$= \frac{3}{4}[64]$$

$$= \frac{48}{1} = 48$$

Think

$$3.5 = 3\frac{1}{2}$$

2 Simplify:  $\frac{[(3.2 + 4.8)^2 + (5 + 1)^2] + 30}{[(7 - 4)^4 - (8.5 - 6.5)^4]}$

$$\frac{[(3.2 + 4.8)^2 + (5 + 1)^2] + 30}{[(7 - 4)^4 - (8.5 - 6.5)^4]} = \frac{[(8)^2 + (6)^2] + 30}{[(3)^4 - (2)^4]}$$

$$= \frac{[64 + 36] + 30}{[81 - 16]}$$

$$= \frac{100 + 30}{65}$$

$$= \frac{130}{65} = 2$$

3 Simplify:  $\frac{4|7 - 11|}{|6 + 2| \cdot |6 - 9|}$

$$\frac{4|7 - 11|}{|6 + 2| \cdot |6 - 9|} = \frac{4|-4|}{|8| \cdot |-3|}$$

$$= \frac{4 \cdot 4}{8 \cdot 3} \quad \leftarrow \text{Calculate within the absolute-value symbols. Next, evaluate the absolute values.}$$

$$= \frac{16}{24} = \frac{2}{3} \quad \leftarrow \text{Multiply, then use common factors to simplify the fraction.}$$

## Try These

Simplify.

1.  $-(-6) + 3(2 + 15) \div \sqrt{9}$

2.  $5^2[-2 + (-5)] \div 5 + 2$

3.  $[(-15.2 + 5.2)^4 + 0.47] - 10^3$

4.  $\frac{6 + 4 \cdot (-2^3)}{2 \cdot 25 + 2} + 2\frac{1}{2}$

5.  $\frac{[(11 - 2)^2 + 3^3 + 12]}{0.5 + 1.5} - 2^4$

6.  $\frac{|-4 + 4(6 - 8)^2| \cdot |(3 - 2 \cdot 8)|}{2^2 \cdot |5 - 2^3|}$

7. **Discuss and Write** Karen simplified the expression  $12^2 \div 2^4 + 9 \div (2 + 1) + 12$ , and her result was 18. Was her answer correct? Explain your answer.

