Name ______________________________

1-9A Sums and Products of Rational and Irrational Numbers

Objective **To investigate why sums or products of two numbers are rational or irrational**

The following rules for integers and irrational numbers can help you investigate the sums or products of rational numbers and irrational numbers.

Key Concept

- The sum, difference, or product of two integers is an integer.
- The sum or difference of an integer and an irrational number is irrational.
- The product or quotient of a nonzero integer and an irrational number is irrational.

Remember
The quotient of two integers is rational if the divisor is not equal to zero.

► To show that the sum of two rational numbers is rational, you can investigate the sum of two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, where a, b, c, and d are integers, $b \neq 0$, and $d \neq 0$.

1 $\frac{a}{b} + \frac{c}{d} = \frac{a(d)}{b(d)} + \frac{(b)c}{(b)d} = \frac{ad + bc}{bd}$ ⟵ Find the sum.

2 ad, bc, and bd are integers. ⟵ The product of two integers is an integer.

3 $ad + bc$ is an integer. ⟵ The sum of two integers is an integer.

4 $\frac{ad + bc}{bd}$ is rational. ⟵ The quotient of two integers is rational if $bd \neq 0$.

Therefore, the sum of two rational numbers is rational.

Think
Let a, b, c, and d be integers and q be any irrational number with $b \neq 0$ and $d \neq 0$.

► To show that the sum of a rational number and an irrational number is irrational, you can investigate whether the sum of a rational number $\frac{a}{b}$ and an irrational number q can be equal to $\frac{c}{d}$, a rational number.

1 $\frac{a}{b} + q = \frac{c}{d}$ ⟵ Start by assuming that this equation is true.

2 $q = \frac{c}{d} - \frac{a}{b} = \frac{(b)c}{(b)d} - \frac{a(d)}{b(d)} = \frac{bc - ad}{bd}$ ⟵ Subtract $\frac{a}{b}$ from both sides; simplify.

3 $\frac{bc - ad}{bd}$ is irrational. ⟵ q is irrational so the expression equal to q is irrational.

4 bc, ad, and bd are integers. ⟵ The product of two integers is an integer.

5 $bc - ad$ is an integer. ⟵ The difference of two integers is an integer.

6 $\frac{bc - ad}{bd}$ is rational. ⟵ The quotient of two integers is rational if $bd \neq 0$.

Step 3 and Step 6 of the proof contradict each other. The expression $\frac{bc - ad}{bd}$ cannot be irrational and rational at the same time. Therefore, the equation in Step 1 cannot be true. $\frac{a}{b} + q$ cannot be rational, so it is irrational.

Tell whether the expression is *rational* or *irrational*.

1. $-\frac{6}{11} + \frac{3}{4}$ ____________ **2.** $\sqrt{10} + \frac{1}{2}$ ____________ **3.** 2π ____________

MATHEMATICAL PRACTICES 2

4. Discuss and Write Explain why the sum of two fractions is rational.

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Name ______________________

1-9A Sums and Products of Rational and Irrational Numbers

Let a, b, c, and d be integers, where $b \neq 0$ and $d \neq 0$. Let q be any irrational number.
Complete the statements to investigate products of rational and irrational numbers.

5. The product of two rational numbers is rational.

a. Find the product: $\frac{a}{b} \bullet \frac{c}{d} =$ __________

b. The __________ of two integers is an integer, so ac and bd are integers.

c. The __________ of two integers is rational, so $\frac{ac}{bd}$ is __________.

d. This shows that the __________ of two rational numbers is __________.

6. The product of a nonzero rational number and an irrational number is irrational.

a. Find the product: $\frac{a}{b} \bullet q =$ __________

b. The __________ of an irrational number and an integer is irrational, so aq is irrational.

c. The __________ of an irrational number and an integer is irrational, so $\frac{aq}{b}$ is irrational.

d. This shows that the product of a nonzero rational number and an irrational number is __________.

Tell whether the expression is *rational* or *irrational*.

7. $\frac{8}{15} + \sqrt{2}$ __________ **8.** $\sqrt{5} \bullet \frac{9}{4}$ __________ **9.** $-\frac{3}{5} + \frac{3}{20}$ __________

10. $\frac{5}{8} + 0.5$ __________ **11.** $\sqrt{16} \bullet \frac{1}{7}$ __________ **12.** $13 + \sqrt{28}$ __________

13. $-\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ __________ **14.** $\frac{3}{8} + \frac{2}{9} + \sqrt{6}$ __________ **15.** $\frac{1}{6} \bullet \frac{\pi}{2} \bullet \frac{4}{15}$ __________

Problem Solving

A counterexample is a single example that shows a statement is false.
Find a counterexample to show that each statement is false.

16. The difference between a rational number and an irrational number is rational.

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17. The product of a rational number and an irrational number is irrational.

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18. The product of two irrational numbers is irrational.

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19. The sum of two irrational numbers is irrational.

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EXPLAIN YOUR REASONING

MATHEMATICAL 3 PRACTICES

20. Explain why the sum of a repeating decimal such as 0.5454545454 . . . and a nonrepeating and nonterminating decimal such as 0.12123123412345 . . . is irrational.

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