1-9

Properties of Real Numbers

Objective To identify the properties of real numbers in addition and multiplication
• To justify the simplification of algebraic expressions by applying the properties of real numbers

The following are properties of real numbers in addition and multiplication.

Let a, b, and c represent real numbers.

Property	Addition	Multiplication
Closure	$a + b$ is a unique real number.	$a \bullet b$ is a unique real number.
Commutative	$a + b = b + a$	$a \bullet b = b \bullet a$
Associative	$a + b + c = (a + b) + c$ $= a + (b + c)$	$a \bullet b \bullet c = (a \bullet b) \bullet c$ $= a \bullet (b \bullet c)$
Identity	$a + 0 = a$ and $0 + a = a$ 0 is the *additive identity* element.	$a \bullet 1 = a$ and $1 \bullet a = a$ 1 is the *multiplicative identity* element.
Inverse	For every real number a, there is a unique real number $-a$ such that $a + (-a) = 0$ and $-a + a = 0$. $-a$ is the *additive inverse* of a, or the *opposite* of a.	For every nonzero real number a, there is a unique real number $\frac{1}{a}$ such that $a \bullet \frac{1}{a} = 1$ and $\frac{1}{a} \bullet a = 1$. $\frac{1}{a}$ is the *multiplicative inverse* of a, or the *reciprocal* of a.
Distributive	$a \bullet (b + c) = a \bullet b + a \bullet c$ and $(b + c) \bullet a = b \bullet a + c \bullet a$ Multiplication is distributive over addition.	

► You can simplify an algebraic expression by applying the properties of real numbers and combining like terms. Terms that have exactly the same literal coefficients that are raised to the same power are called like terms.

Like terms: $2a$ and a
xy^2 and $-2xy^2$

Unlike terms: mn and $-3mn^2$

Think
The variables are *not raised to the same powers*.

Use the properties of real numbers to justify the steps of a simplification process.

Simplify: $3(m + 9) + 2m$

$[3(m) + 3(9)] + 2m$ ← Apply the Distributive Property.

$3m + 27 + 2m$ ← Multiply.

$3m + 2m + 27$ ← Apply the Commutative Property to get like terms near each other.

$(3m + 2m) + 27$ ← Apply the Associative Property to group the like terms.

$(3 + 2)m + 27$ ← Apply the Distributive Property to combine like terms.

$5m + 27$ ← Add the coefficients of the like terms.

Examples

1 Simplify: $7(y + 2) - 5(y + 8)$

$7(y + 2) + (-5)(y + 8)$ ← Apply the definition of subtraction: $a - b = a + (-b)$

$[7(y) + 7(2)] + [(-5)(y) + (-5)(8)]$ ← Apply the Distributive Property.

$[7y + 14] + [(-5y) + (-40)]$ ← Multiply.

$[7y + 14 + (-5y)] + (-40)$ ← Apply the Associative Property.

$[7y + (-5y) + 14] + (-40)$ ← Apply the Commutative Property.

$[7y + (-5y)] + [14 + (-40)]$ ← Apply the Associative Property.

$[7 + (-5)]y + [14 + (-40)]$ ← Apply the Distributive Property.

$2y + (-26)$ ← Combine like terms.

$2y - 26$ ← Apply the definition of subtraction: $a + (-b) = a - b$

2 Simplify: $2z - (z + 3)$

$2z - 1(z + 3)$ ← Apply the Identity Property for Multiplication.

$2z + (-1)(z + 3)$ ← Apply the definition of subtraction.

$2z + [(-1 \bullet z) + (-1 \bullet 3)]$ ← Apply the Distributive Property.

$2z + [(-1z) + (-3)]$ ← Multiply.

$[2z + (-1z)] + (-3)$ ← Apply the Associative Property.

$[2 + (-1)]z + (-3)$ ← Apply the Distributive Property.

$1z + (-3)$ ← Combine like terms.

$z - 3$ ← Apply the Identity Property for Multiplication and the definition of subtraction.

Try These

Substitute a number for *n* to make each statement true. Identify the property or definition that is illustrated.

1. $7(10 + 1) = 7(10) + 7n$

2. $8 + n = 9 + 8$

3. $(3 + 4) + 8 = 3 + (n + 8)$

4. $7n = 1$

5. $9 \bullet \frac{1}{9} = n$

6. $n \bullet 12 = 12$

7. $6 + n = 0$

8. $16 - (-5) = 16 + n$

9. $8 \div 4 = 8n$

Write a justification for each step of the given simplification process.

10. Simplify: $7w - 5(3 + w)$

a. $7w + (-5)(3 + w)$

b. $7w + (-5)(3) + (-5)(w)$

c. $7w + (-15) + (-5w)$

d. $7w + (-5w) + (-15)$

e. $[7w + (-5w)] + (-15)$

f. $[7 + (-5)]\ w + (-15)$

g. $2w + (-15)$

h. $2w - 15$

11. Discuss and Write Explain how to simplify $4x + 6y + 3x - 2y + 8$. Show all steps.

Go to PRACTICE BOOK Lesson 1-9 for exercise sets.