2-1 Open Sentences and Solution Sets

Objective To distinguish between open and closed number sentences • To determine whether a given number is a solution to a given equation • To solve equations by substitution, given replacement sets

A theater stage crew needs to fill a triangular gap in the play's set. The area of the space is 14 ft^2, the base is 7 ft, and the height is not given. What mathematical sentence can you use to determine the height of the space?

You can use the formula $A = \frac{1}{2}bh$ that relates the area of a triangle to the length of a base, b, and the corresponding height, h.

Let h = the height of the triangular figure.

$\text{area} = \frac{1}{2} \bullet \text{base} \bullet \text{height}$

$14 = \frac{1}{2} \bullet 7h$ ← Substitute the known values.

The algebraic equation, $14 = \frac{1}{2} \bullet 7h$ shows that the algebraic expression $\frac{1}{2} \bullet 7h$ *is equal to* the numerical expression 14.

So the mathematical sentence for the area of the space is $14 = \frac{1}{2} \bullet 7h$.

Key Concept

Equation

An equation is a statement that two mathematical expressions are equal.

► Equations can be either *open sentences* or *closed sentences*.
Open sentences contain variables that represent the unknown quantities and are neither true nor false.
Closed sentences contain no variables and are either true or false.

Open Sentence	Closed Sentence
$14 = \frac{1}{2} \bullet 7h$	$14 = \frac{1}{2} \bullet 7 \bullet 4$
	$14 = \frac{1}{2} \bullet 28$
	$14 = 14$ True

Think
This closed sentence is true because 4 *satisfies* the equation.

The number 4 is the solution to the equation. Solution set: {4}.

Examples

Determine whether the given value of the variable makes the open sentence *true* or *false*.

1 $5(m + 7) = 5m + 7$ when $m = 3$

$5(m + 7) = 5m + 7$

$5(3 + 7) \stackrel{?}{=} 5(3) + 7$

$5(10) \stackrel{?}{=} 15 + 7$

$50 = 22$ False

2 $3k^2 + 18 = 30$ when $k = -2$

$3k^2 + 18 = 30$

$3(-2)^2 + 18 \stackrel{?}{=} 30$

$3(4) + 18 \stackrel{?}{=} 30$

$12 + 18 \stackrel{?}{=} 30$

$30 = 30$ True

▶ To determine the solution set for an open sentence, given a replacement set, R, or domain set, you must substitute every element in the replacement set into the equation. That is, replace the variable with each element in that set.

A *replacement set* is the set of elements that can be substituted for a variable.

Using the replacement set $\{-2, 0, 2\}$, find the solution set for the open sentence $|3c^2 - 1| = 11$.

Substitute each element of the replacement set into the equation. After replacing the variable c with these elements, evaluate the resulting closed sentences and determine if they are *true* or *false*.

$|3(-2)^2 - 1| \stackrel{?}{=} 11$
$|3(4) - 1| \stackrel{?}{=} 11$
$|12 - 1| \stackrel{?}{=} 11$
$11 = 11$ True

$|3(0)^2 - 1| \stackrel{?}{=} 11$
$|3(0) - 1| \stackrel{?}{=} 11$
$|0 - 1| \stackrel{?}{=} 11$
$1 = 11$ False

$|3(2)^2 - 1| \stackrel{?}{=} 11$
$|3(4) - 1| \stackrel{?}{=} 11$
$|12 - 1| \stackrel{?}{=} 11$
$11 = 11$ True

So the solution set is $\{-2, 2\}$; this can be also written as $\{\pm 2\}$.

Think
Only the numbers from the replacement set that make the open sentence *true* will be in the solution set.

▶ It is possible that no element in a replacement set will make an open sentence true. If this is the case, then the solution set of the equation is the empty set, or null set, which is represented by the symbol $\varnothing$ or $\{\ \}$.

Using the replacement set $\{-1, 1, 2\}$, find the solution set for the open sentence $-4a^2 = (-4a)^2$.

$-4(-1)^2 \stackrel{?}{=} [-4(-1)]^2$
$-4(1) \stackrel{?}{=} (4)^2$
$-4(1) \stackrel{?}{=} 16$
$-4 = 16$ False

$-4(1)^2 \stackrel{?}{=} [-4(1)]^2$
$-4(1) \stackrel{?}{=} (-4)^2$
$-4(1) \stackrel{?}{=} 16$
$-4 = 16$ False

$-4(2)^2 \stackrel{?}{=} [-4(2)]^2$
$-4(4) \stackrel{?}{=} (-8)^2$
$-4(4) \stackrel{?}{=} 64$
$-16 = 64$ False

So the solution set is $\varnothing$ or $\{\ \}$.

Try These

Identify each as an *open sentence*, a *true sentence*, a *false sentence*, or an *expression*.

1. $2^5 = 5^2$ **2.** $-7x^2$ **3.** $2x + 5 = 13$ **4.** $8 + 3(2) = 14$ **5.** $10 = \frac{1}{2}(8x)$

Using the replacement set $\{-2, -1, 1.2, 2, 4\}$, find the solution set for each.

6. $d + 5 = 9$ **7.** $8 = 3g - 4$ **8.** $2k - 3.6 = 0.2k$ **9.** $-b^2 + 11 = 1.5$ **10.** $n^2 = (-n)^2$

11. Discuss and Write Soup is on sale at \$0.99 per can. The number of cans a customer can buy is limited to 5. Write an algebraic expression to model the cost of a number of cans of this soup. Tell what the variable represents, and write a domain set for that variable.

Go to PRACTICE BOOK Lesson 2-1 for exercise sets.