2-3 Solve Multiplication and Division Equations

Objective To solve and check multiplication equations using the Division Property of Equality • To solve and check division equations using the Multiplication Property of Equality • To justify the steps of the solution process for multiplication and division equations

Raul's dad is helping to build props for a school play. He needs to cut a 2-foot-long wooden board into 4 short boards of equal length. What will be the measure of each short board?

▶ To find the measure of each short board, write and solve an equation. First, identify the variable. Then be sure that all measures are expressed with the same unit of measure.

Let x = the length of each of the short boards Raul's dad cuts, in inches.

$4x$ ← the expression for the total length of the short boards, in inches

$2 \text{ ft} = 2(12 \text{ in.}) = 24 \text{ in.}$ ← the expression for the length of the long board, in inches

$4x = 24$ ← an algebraic equation showing equal expressions for the length of the long board, in inches

Remember: 1 foot = 12 inches

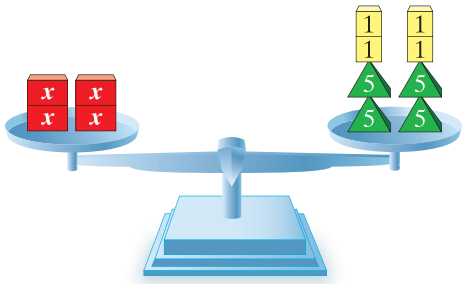
Key Concept

Division Property of Equality

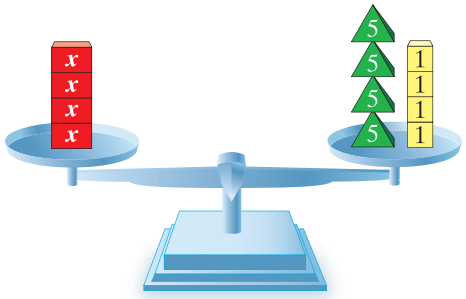
For real numbers, a, b, and c, $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

▶ An equation that contains only the operation of multiplication, such as $4x = 24$, is called a multiplication equation. To solve a multiplication equation, use the Division Property of Equality.

You can use a balance to visualize how a multiplication equation is solved.



$4x = 24$



$\frac{4x}{4} = \frac{24}{4}$ ← Use the Division Property of Equality.

$x = 6$ ← equivalent equation

So the length of each short board that Raul's dad cuts is 6 inches.

▶ To solve algebraically a multiplication equation, isolate the unknown term by using division.

Solve: $-8.5 = 5y$

$\frac{-8.5}{5} = \frac{5y}{5}$ ← Use the Division Property of Equality.

$-1.7 = y$ ← solution

Solution set: $\{-1.7\}$

Check: $-8.5 = 5y$

$-8.5 \stackrel{?}{=} 5 \bullet 1.7$

$-8.5 = -8.5$ True

Example

1 **Solve:** $-5\frac{3}{8} = 2q$

$\frac{-5\frac{3}{8}}{2} = \frac{2q}{2}$ ← Use the Division Property of Equality.

$-\frac{43}{8} \div \frac{2}{1} = q$ ← Simplify.

$-\frac{43}{8} \bullet \frac{1}{2} = q$

$-\frac{43}{16} = -2\frac{11}{16} = q$ ← solution

Solution set: $\left\{-2\frac{11}{16}\right\}$

Check: $-5\frac{3}{8} = 2q$

$-\frac{43}{8} \stackrel{?}{=} 2\left(-2\frac{11}{16}\right)$

$-\frac{43}{8} \stackrel{?}{=} 2\left(-\frac{43}{16}\right)$

$-\frac{43}{8} = -\frac{43}{8}$ True

► An equation that contains only the operation of division is called a division equation. To solve a division equation, use the Multiplication Property of Equality.

Key Concept

Multiplication Property of Equality

For real numbers, a, b, and c, if $a = b$, *then* $ac = bc$.

A box of paperback books is divided among 11 high school students. If each student receives 3 books, how many paperbacks were in the box?

To find how many paperbacks, write and solve a division equation.

Let b = the number of paperback books in the box.

number of paperbacks in a box — *divided by* — eleven — *equals* — number of paperbacks per child

$b \quad \div \quad 11 \quad = \quad 3$ ← division equation

Think $b \div 11 = \frac{b}{11}$

Solve: $\frac{b}{11} = 3$

$\frac{b}{11} \bullet 11 = 3 \bullet 11$ ← Use the Multiplication Property of Equality.

$b = 33$ ← solution

So the box contained 33 paperbacks.

Check: $\frac{b}{11} = 3$

$\frac{33}{11} \stackrel{?}{=} 3$

$3 = 3$ True

► In some equations, the coefficient of the unknown term will be a fraction. In such cases, multiply by the reciprocal of the coefficient to isolate the unknown term.

Remember: The *reciprocal* of a number is formed by interchanging the numerator and denominator.

Solve: $\frac{3}{7}z = 36$

$\frac{7}{3} \bullet \frac{3}{7}z = \frac{7}{3} \bullet 36$ ← Multiply each side by the reciprocal of $\frac{3}{7}$.

$z = \frac{7}{\cancel{3}_{1}} \bullet \frac{\overset{12}{\cancel{36}}}{1}$ ← Simplify.

$z = 84$ ← solution

Solution set: {84}

Check: $\frac{3}{7}z = 36$

$\frac{3}{7}(84) \stackrel{?}{=} 36$

$36 = 36$ True

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Example

1 **Solve:** $\frac{2a}{3} = \frac{4}{9}$

Think $\frac{2a}{3}$ is equivalent to $\frac{2}{3}a$.

$\frac{2}{3}a = \frac{4}{9}$

$\frac{3}{2} \cdot \frac{2}{3}a = \frac{3}{2} \cdot \frac{4}{9}$ ← Use the Multiplication Property of Equality.

$a = \frac{\overset{1}{\cancel{3}}}{\underset{1}{2}} \cdot \frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{9}}} = \frac{2}{3}$ ← Simplify.

$a = \frac{2}{3}$ ← solution

Solution set: $\{\frac{2}{3}\}$

Check: $\frac{2}{3}a = \frac{4}{9}$

$\frac{2}{3}(\frac{2}{3}) \stackrel{?}{=} \frac{4}{9}$

$\frac{4}{9} = \frac{4}{9}$ True

You can use the Multiplication Property of Equality to solve a multiplication equation or a division equation. Along with other familiar properties and definitions, you can justify the steps of the solution process.

Solve: $6x = 72$. Write a justification for each step in the solution process.

$\frac{1}{6} \cdot 6x = \frac{1}{6} \cdot 72$ ← Use the Multiplication Property of Equality.

$(\frac{1}{6} \cdot 6) \cdot x = \frac{1}{6} \cdot 72$ ← Use the Associative Property of Multiplication.

$1 \cdot x = \frac{1}{6} \cdot 72$ ← Use the Multiplicative Inverse Property.

$x = \frac{1}{6} \cdot 72$ ← Use the Multiplicative Identity Property.

$x = 12$ ← solution

Solution set: {12}

Check: $6x = 72$

$6(12) \stackrel{?}{=} 72$

$72 = 72$ True

Examples

1 **Solve:** $\frac{k}{4} = 16$

$k \cdot \frac{1}{4} = 16$ ← Use the definition of division.

$k \cdot \frac{1}{4} \cdot 4 = 16 \cdot 4$ ← Use the Multiplication Property of Equality.

$k \cdot (\frac{1}{4} \cdot 4) = 16 \cdot 4$ ← Use the Associative Property of Multiplication.

$k \cdot 1 = 16 \cdot 4$ ← Use the Multiplicative Inverse Property.

$k = 16 \cdot 4$ ← Use the Multiplicative Identity Property.

$k = 64$ ← Simplify.

Solution set: {64}

Check: $\frac{k}{4} = 16$

$\frac{64}{4} \stackrel{?}{=} 16$

$16 = 16$ True

2 **Solve:** $11\frac{1}{4} + 3.75 = 5x$

$11\frac{1}{4} + 3.75 = 5x$ ← Identify like terms.

$15 = 5x$ ← Combine like terms.

$\frac{15}{5} = \frac{5 \cdot x}{5}$ ← Use the Division Property of Equality.

$3 = 1 \cdot x$ ← Simplify.

$3 = x$ ← Use the Multiplicative Identity Property.

Think $11\frac{1}{4} = 11.25$

Solution set: {3}

Check: $11\frac{1}{4} + 3.75 = 5x$

$11\frac{1}{4} + 3.75 \stackrel{?}{=} 5(3)$

$15 = 15$ True

3 **Solve:** $19h = -114$

$\frac{19h}{19} = \frac{-114}{19}$ ← Use the Division Property of Equality.

$\frac{19h}{19 \cdot 1} = \frac{-114}{19}$ ← Use the Multiplicative Identity Property.

$\frac{19}{19} \cdot \frac{h}{1} = \frac{-114}{19}$ ← $\frac{ac}{bd} = \frac{a}{b} \cdot \frac{b}{d}$

$1 \cdot h = \frac{-114}{19}$ ← $\frac{a}{a} = 1; \frac{a}{1} = a$

$h = \frac{-114}{19}$ ← Use the Multiplicative Identity Property.

$h = -6$ ← Simplify.

Solution set: {−6}

Check: $19h = -114$

$19(-6) \stackrel{?}{=} -114$

$-114 = -114$ True

Try These

Solve each equation. Check your solution.

1. $-10.2 = 1.7j$ **2.** $-80 = \frac{h}{5}$ **3.** $2b = -7\frac{3}{4}$ **4.** $\frac{-4n}{5} = \frac{16}{25}$

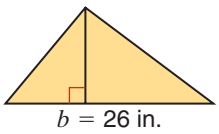
Solve the equation. Write a justification for each step. Check your solution.

5. $27 = \frac{z}{9}$ **6.** $-216 = 72g$ **7.** $\frac{2}{5}z = 10$ **8.** $\frac{7}{2}m = 12.1 + 8.9$

Write and solve an equation.

9. The tallest player on a basketball team is $72\frac{3}{4}$ inches tall. If this is $1\frac{1}{4}$ times the height of the shortest player, what is the height of the shortest player?

10. The area of the triangle at the right is 156 in^2. If the base is 26 in. long, write and solve an equation that can be used to find the height h.



Hint: Area $\triangle = \frac{1}{2}bh$

11. **Discuss and Write** To solve the equation $\frac{x}{8} = 56$, Kyle divided 56 by 8 and said the solution is $x = 7$. Do you agree or disagree? Explain.

Go to PRACTICE BOOK Lesson 2-3 for exercise sets.