2-4 Solve Equations with Two Operations

Objective To solve and check a variety of equations with two operations

PROBLEM-SOLVING STRATEGY 1

At a fair to promote recycling, a man used empty cans to make a costume. The number of cans he used was 5 more than twice the number of cans that had been in one collection bin. If there were 105 cans in the costume, how many cans were in the collection bin?

To find how many cans were in the bin, write and solve an equation.

Let x = the number of cans in the bin.

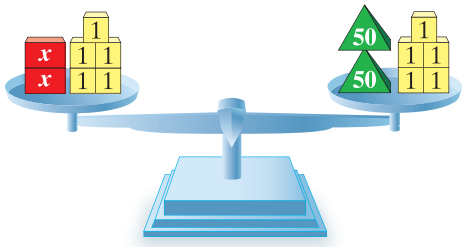
five	*more than*	twice the number of cans	*equals*	number of cans in the costume
5	+	$2x$	=	105 ← equation with two operations

▶ To solve an *equation with two operations*, isolate the variable by using inverse operations.

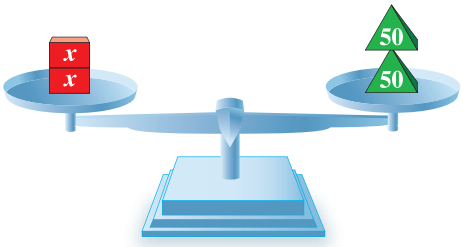
You can use a balance to visualize an equation with two operations and its solution.

Solve: $2x + 5 = 105$

Think $5 + 2x = 105$ is equivalent to $2x + 5 = 105$.



$2x + 5 = 105$



$2x + 5 - 5 = 105 - 5$ ← Subtract 5 from both sides.

$\frac{2x}{2} = \frac{100}{2}$ ← Divide both sides by 2.

$x = 50$

So there were 50 cans in the collection bin.

Examples

1 **Solve:** $\frac{k}{3} - 0.5 = 4.5$

$\frac{k}{3} - 0.5 + 0.5 = 4.5 + 0.5$ ← Use the Addition Property of Equality.

$\frac{k}{3} = 5$ ← Simplify.

$3 \cdot \frac{k}{3} = 3 \cdot 5$ ← Use the Multiplication Property of Equality.

$k = 15$ ← Simplify.

Solution set: {15}

Check: $\frac{k}{3} - 0.5 = 4.5$

$\frac{15}{3} - 0.5 \stackrel{?}{=} 4.5$

$5 - 0.5 \stackrel{?}{=} 4.5$

$4.5 = 4.5$ True

2 **Solve:** $11 - b = 32$

$11 - 1b = 32$ ← The coefficient of b is 1.

$11 + (-1b) = 32$ ← Use the definition of subtraction.

$11 - 11 + (-1b) = 32 - 11$ ← Use the Subtraction Property of Equality.

$-1b = 21$ ← Simplify.

$\frac{-1b}{-1} = \frac{21}{-1}$ ← Use the Division Property of Equality.

$b = -21$ ← Simplify.

Solution set: $\{-21\}$

Check: $11 - b = 32$

$11 - (-21) \stackrel{?}{=} 32$

$11 + 21 \stackrel{?}{=} 32$

$32 = 32$ True

▶ When an equation contains several fractions, you can either compute with fractions throughout, *or* you can transform the equation to an equivalent equation with integer coefficients.

Solve: $\frac{w}{18} - \frac{1}{9} = \frac{7}{9}$

Method 1 Compute with fractions.

$\frac{w}{18} - \frac{1}{9} = \frac{7}{9}$

$\frac{w}{18} - \frac{1}{9} + \frac{1}{9} = \frac{7}{9} + \frac{1}{9}$ ← Use the Addition Property of Equality.

$\frac{w}{18} = \frac{8}{9}$ ← Simplify.

$18 \cdot \frac{w}{18} = 18 \cdot \frac{8}{9}$ ← Use the Multiplication Property of Equality.

$w = \cancel{18}^{2} \cdot \frac{8}{\cancel{9}_{1}} = 16$ ← Simplify.

Method 2 Use the LCD to transform the equation.

$\frac{w}{18} - \frac{1}{9} = \frac{7}{9}$ ← Find the LCD of 9 and 18. LCD = 18

$18\left(\frac{w}{18} - \frac{1}{9}\right) = 18 \cdot \frac{7}{9}$ ← Multiply each side of the equation by the LCD.

$18 \cdot \frac{w}{18} - 18 \cdot \frac{1}{9} = 18 \cdot \frac{7}{9}$ ← Distribute 18 on the left side.

$w - 2 = 14$ ← Simplify.

$w - 2 + 2 = 14 + 2$ ← Use the Addition Property of Equality.

$w = 16$ ← Simplify.

Solution set: $\{16\}$

Check: $\frac{w}{18} - \frac{1}{9} = \frac{7}{9}$

$\frac{16}{18} - \frac{1}{9} \stackrel{?}{=} \frac{7}{9}$

$\frac{8}{9} - \frac{1}{9} \stackrel{?}{=} \frac{7}{9}$

$\frac{7}{9} = \frac{7}{9}$ True

Continue Lesson ➡

▶ If the unknown appears in more than one term, you will need to first combine like terms, then solve.

Remember: *Like terms* have the same variable raised to the same power.

Examples

1 **Solve:** $2p - 7p - 1.98 = -35$

$2p - 7p - 1.98 = -35$ ← Identify like terms.

$-5p - 1.98 = -35$ ← Combine like terms.

$-5p - 1.98 + 1.98 = -35 + 1.98$ ← Use the Addition Property of Equality.

$-5p = -33.02$ ← Simplify.

$\frac{-5p}{-5} = \frac{-33.02}{-5}$ ← Use the Division Property of Equality.

$p = 6.604$ ← Simplify.

Check: $2p - 7p - 1.98 = -35$

$2(6.604) - 7(6.604) - 1.98 \stackrel{?}{=} -35$

$13.208 - 46.228 - 1.98 \stackrel{?}{=} -35$

$-33.02 - 1.98 \stackrel{?}{=} -35$

$-35 = -35$ True

Solution set: $\{6.604\}$

2 **Solve:** $7y - 1.8y + 4.3 = -8 + 7.1$

$7y - 1.8y + 4.3 = -8 + 7.1$ ← Identify like terms.

$5.2y + 4.3 = -0.9$ ← Combine like terms.

$5.2y + 4.3 - 4.3 = -0.9 - 4.3$ ← Use the Subtraction Property of Equality.

$5.2y = -5.2$ ← Simplify.

$\frac{5.2y}{5.2} = \frac{-5.2}{5.2}$ ← Use the Division Property of Equality.

$y = -1$ ← Simplify.

Check: $7y - 1.8y + 4.3 = -8 + 7.1$

$7(-1) - 1.8(-1) + 4.3 \stackrel{?}{=} -8 + 7.1$

$-7 + 1.8 + 4.3 \stackrel{?}{=} -8 + 7.1$

$-0.9 = -0.9$ True

Solution set: $\{-1\}$

3 There are three consecutive integers in counting order whose sum is 84. What are the integers?

Let x, $x + 1$, and $x + 2$ represent the three consecutive integers.

Solve: $x + (x + 1) + (x + 2) = 84$

$x + x + 1 + x + 2 = 84$ ← Identify like terms.

$3x + 3 = 84$ ← Combine like terms.

$3x + 3 - 3 = 84 - 3$ ← Use the Subtraction Property of Equality.

$3x = 81$ ← Simplify.

$\frac{3x}{3} = \frac{81}{3}$ ← Use the Division Property of Equality.

$x = 27$ ← Simplify.

$x + 1 = 27 + 1 = 28$

$x + 2 = 27 + 2 = 29$ ← Find the other integers.

So the consecutive integers are 27, 28, and 29.

Check: $x + (x + 1) + (x + 2) = 84$

$27 + 28 + 29 \stackrel{?}{=} 84$

$84 = 84$ True

You can use the Properties of Equality along with other familiar properties and definitions to justify the steps of a solution process when solving an equation involving more than one operation.

Solve: $7 - 0.8x - 11.7 + 0.1x = 2.3$

$(0.1x - 0.8x) + (7 - 11.7) = 2.3$ ← Use the Commutative and Associative Properties to collect like terms.

$(0.1 - 0.8)x + (7 - 11.7) = 2.3$ ← Use the Distributive Property on the x-terms.

$-0.7x + (-4.7) = 2.3$ ← Simplify.

$-0.7x - 4.7 = 2.3$ ← Use the definition of subtraction.

$-0.7x - 4.7 + 4.7 = 2.3 + 4.7$ ← Use the Addition Property of Equality.

$-0.7x = 7$ ← Simplify.

$\frac{1}{-0.7} \cdot -0.7x = \frac{1}{-0.7} \cdot 7$ ← Use the Multiplication Property of Equality.

$\left(-0.7 \cdot \frac{1}{-0.7}\right)x = \frac{1}{-0.7} \cdot 7$ ← Use the Associative Property of Multiplication.

$(1)x = \frac{1}{-0.7} \cdot 7$ ← Use the Multiplicative Inverse Property.

$x = -10$

Check:

$7 - 0.8x - 11.7 + 0.1x = 2.3$

$7 - 0.8(-10) - 11.7 + 0.1(-10) \stackrel{?}{=} 2.3$

$7 + 8 - 11.7 - 1 \stackrel{?}{=} 2.3$

$2.3 = 2.3$ True

Try These

Solve each equation. Check your solution.

1. $121.3 = 6x - 42.5$

2. $\frac{2h}{3} - 4 = 14$

3. $12 - 0.5j = -13$

4. $\frac{z}{24} + \frac{1}{12} = \frac{5}{12}$

5. $-n + 8.4 = -9 + 7.7$

6. $2p + p + 4.3 = 8 + 2$

Solve each equation. Write a justification for each step. Check your solution.

7. $38m - 4 = 15$

8. $6y + 3 - 4y = 15$

9. $5 - 8t - 7 - t = 16$

Write and solve an equation for each problem.

10. There are three *consecutive even* integers in counting order whose sum is 102. Find these integers.

11. The measure of the larger acute angle of a right triangle is twice the measure of the smaller acute angle. Find each measure.

12. **Discuss and Write** Compare the process of solving an algebraic equation that contains two operations to the process of evaluating a numerical expression that contains two operations. Include specific examples.

Go to PRACTICE BOOK Lesson 2-4 for exercise sets.