2-5A Solve Equations with Letter Coefficients

Objective To solve linear equations with coefficients represented by letters

The schedule shows the times that four different trains leave the same station and the average speed of each train. Each train follows the same route. How many hours does it take for each train to catch up with the train before it?

Train Schedule		
Train	Departure Time	Average Speed (km/h)
N	12:00	100
R	1:00	120
Q	1:30	125
F	1:45	140

▶ To find these times, you can write and solve an equation using letter coefficients to represent the problem situation. Then you can use that solution to find the times for all the trains.

• Use letters to represent the speeds and times for the trains.

  Let a = the speed of a train
    b = the speed of the next train
    c = the head start (the difference in departure times of the trains)
    x = the unknown time, in hours

Remember
distance = speed × time

Think
• The distances two trains travel are the same.
• The time each train travels is equal to the unknown time minus the head start.

• Write an equation to represent the situation for any two trains.

  distance traveled by first train = distance traveled by second train
  $$ax = b(x - c)$$

• Solve the equation for x.

  $ax = b(x - c)$ ⟵ Write the equation.
  $ax = bx - bc$ ⟵ Use the Distributive Property.
  $ax - bx = bx - bc - bx$ ⟵ Use the Subtraction Property of Equality.
  $ax - bx = -bc$ ⟵ Simplify.
  $(a - b)x = -bc$ ⟵ Use the Distributive Property.
  $\frac{(a - b)x}{(a - b)} = \frac{-bc}{(a - b)}$ ⟵ Use the Division Property of Equality.

  $x = \frac{-bc}{(a - b)}$ ⟵ The result gives the value of x for any a, b, and c.

• Use the solution.

  Replace the letter coefficients with information from the schedule to calculate the time it takes for each train to catch up to the train before it. A table helps organize your work.

Time for Second Train to Overtake First Train				
Trains	Speed of Train, a (km/h)	Speed of Next Train, b (km/h)	Head Start, c (h)	$x = \frac{-bc}{(a - b)}$
N, R	100	120	1	6
R, Q	120	125	0.5	12.5
Q, F	125	140	0.25	$2.\overline{3}$

  So, it takes Train R 6 hours to catch up to Train N, Train Q 12.5 hours to catch up to Train R, and Train F $2.\overline{3}$ hours to catch up to Train Q.

Solve each equation for x.

1. $ax + b = cx + d$ _____

2. $mx = p + nx$ _____

3. Discuss and Write Describe a method for solving a multistep equation in one variable with coefficients represented by letters.

_____

Solve each equation for _y_.

4. $ay - b = c$ _____

5. $(y - k)l = m$ _____

6. $f - gy = (h - j)x$ _____

7. $py + q = rx - sy + t$ _____

Solve each equation for _x_. Then use the solution to complete the table of values.

8. $ax + b = cx$ _____

9. $(k - x)l = m$ _____

a	b	c	x
1	1	0	
2	3	3	
3	5	6	
4	7	9	
5	9	12	

k	l	m	x
5	10	8	
4	5	9	
2	−5	11	
1	−10	12	
0	−15	13	

Problem Solving

Write and solve an equation. Choose values for the letters and evaluate.

10. Bobbie is _b_ years old and her younger brother, Andrew, is _a_ years old. How many years older is Bobbie than Andrew?

_____

11. One building is _c_ meters tall. Another building is _d_ times as tall as the first building. How tall is the other building?

_____

12. Julio went to the beach from his house by walking _l_ km and then riding a bicycle at an average speed of _m_ km/h. He went home by riding the bicycle at an average speed of _n_ km/h. How long did it take Julio to get home if his time spent riding the bicycle was the same both ways?

_____

13. Jane walks _d_ miles around a park at a speed of _a_ mi/h. Bob bikes around the park at a speed _b_ times Jane's speed. Roger runs around the park in the difference between Jane's time and Bob's time. How much time does it take Roger to run around the park? (Hint: Use distance equals rate times time to find Jane and Bob's times.)

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EXPLAIN YOUR REASONING

14. Cathy said that solving an equation with letter coefficients is easier than solving an equation with numerical coefficients. Tell why you agree or disagree with her.

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