2-5 Solve Multistep Equations

Objective To use the Distributive Property in equations with grouping symbols • To solve equations that contain variable terms on both sides • To recognize identities and contradictions • To state general principles for solving an equation in one variable

Nick, Joan, and Maya are selling handmade pottery pieces. So far, Joan has sold 10 more pottery pieces than Nick. Maya has sold twice as many pieces as Joan. If Maya has sold 40 pottery pieces, how many pieces has Nick sold?

► To find how many Nick has sold, write and solve an equation.

Let n = the number of pottery pieces Nick has sold.

$n + 10$ = the number of pottery pieces Joan has sold

twice as the number of pottery pieces Joan sold	*equals*	number of pottery pieces Maya has sold
$2(n + 10)$	$=$	40 ← equation with grouping symbols

► To solve an *equation with grouping symbols*, you can begin by using the Distributive Property.

Solve: $2(n + 10) = 40$

$2(n + 10) = 40$

$2n + 2(10) = 40$ ← Use the Distributive Property.

$2n + 20 = 40$ ← Simplify.

$2n + 20 - 20 = 40 - 20$ ← Use the Subtraction Property of Equality.

$2n = 20$ ← Simplify.

$\frac{2n}{2} = \frac{20}{2}$ ← Use the Division Property of Equality.

$n = 10$ ← solution

Use the value for n to evaluate the other expressions.
Check with the words of the problem to verify the solution.

$n = 10$ ← number of pottery pieces Nick has sold

$n + 10 = 10 + 10 = 20$ ← number of pottery pieces Joan has sold

$2(n + 10) = 2(10 + 10) = 2(20) = 40$ ← number of pottery pieces Maya has sold, which matches the given information

Check: $2(n + 10) = 40$

$2(10 + 10) \stackrel{?}{=} 40$

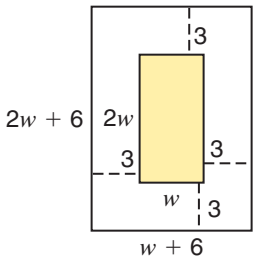
$2(20) \stackrel{?}{=} 40$

$40 = 40$ True

So Nick has sold 10 handmade pottery pieces.

Example

1 Sam framed a picture that is twice as long as it is wide. He used a 3-inch-wide mat around the picture. If Sam used 78 inches of wood to frame the picture, what are the dimensions of the picture?



Let w = the width of the picture, in inches;
$2w$ = the length of the picture, in inches;
$w + 6$ = the width of the frame, in inches; and
$2w + 6$ = the length of the frame, in inches.

Solve: $2(w + 6) + 2(2w + 6) = 78$ ← Use the perimeter formula: $P = 2\ell + 2w$

$2w + 12 + 4w + 12 = 78$ ← Use the Distributive Property twice, and identify like terms.

$6w + 24 = 78$ ← Combine like terms.

$6w + 24 - 24 = 78 - 24$ ← Use the Subtraction Property of Equality.

$6w = 54$ ← Simplify.

$\frac{6w}{6} = \frac{54}{6}$ ← Use the Division Property of Equality.

$w = 9$ ← Simplify.

$2w = 2(9) = 18$

Check: The dimensions of the frame are $w + 6 = 9 + 6 = 15$ and $2w + 6 = 18 + 6 = 24$.

The amount of framing is $2(15) + 2(24) = 30 + 48 = 78$.

So the dimensions of the picture are 9 inches by 18 inches.

▶ To solve an equation that has unknown terms on both sides, use inverse operations to write an equivalent equation with all of the unknown terms on one side.

Solve: $12z - 3 = 8z + 13$

It is usually easier to collect the unknown terms on the side where the resulting coefficient will be positive.

$12z - 3 = 8z + 13$

$12z - 8z - 3 = 8z - 8z + 13$ ← Use the Subtraction Property of Equality.

$4z - 3 = 13$ ← Simplify.

$4z - 3 + 3 = 13 + 3$ ← Use the Addition Property of Equality.

$4z = 16$ ← Simplify.

$\frac{4z}{4} = \frac{16}{4}$ ← Use the Division Property of Equality.

$z = 4$ ← Simplify.

Solution set: {4}

Check: $12z - 3 = 8z + 13$

$12(4) - 3 \stackrel{?}{=} 8(4) + 13$

$48 - 3 \stackrel{?}{=} 32 + 13$

$45 = 45$ True

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▶ Before collecting unknown terms, you may first need to simplify the equation.

Solve: $6v + v + 9 = 4 - 3v$

$6v + v + 9 = 4 - 3v$ ← Identify like terms.

$7v + 9 = 4 - 3v$ ← Combine like terms.

$7v + 3v + 9 = 4 - 3v + 3v$ ← Use the Addition Property of Equality.

$10v + 9 = 4$ ← Simplify.

$10v + 9 - 9 = 4 - 9$ ← Use the Subtraction Property of Equality.

$10v = -5$ ← Simplify.

$\frac{10v}{10} = \frac{-5}{10}$ ← Use the Division Property of Equality.

$v = -\frac{1}{2}$

Solution set: $\left\{-\frac{1}{2}\right\}$

Check: $6v + v + 9 = 4 - 3v$

$6\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right) + 9 \stackrel{?}{=} 4 - 3\left(-\frac{1}{2}\right)$

$-3 + \left(-\frac{1}{2}\right) + 9 \stackrel{?}{=} 4 + \frac{3}{2}$

$-3\frac{1}{2} + 9 \stackrel{?}{=} 4 + 1\frac{1}{2}$

$5\frac{1}{2} = 5\frac{1}{2}$ True

Example

1 **Solve:** $2(3m - 1) = 2m + 2$

$2(3m - 1) = 2m + 2$

$2(3m) + 2(-1) = 2m + 2$ ← Use the Distributive Property.

$6m - 2 = 2m + 2$ ← Simplify.

$6m - 2m - 2 = 2m - 2m + 2$ ← Use the Subtraction Property of Equality.

$4m - 2 = 2$ ← Simplify.

$4m - 2 + 2 = 2 + 2$ ← Use the Addition Property of Equality.

$4m = 4$ ← Simplify.

$\frac{4m}{4} = \frac{4}{4}$ ← Use the Division Property of Equality.

$m = 1$

Solution set: $\{1\}$

Check: $2(3m - 1) = 2m + 2$

$2(3 \cdot 1 - 1) \stackrel{?}{=} 2 \cdot 1 + 2$

$2(3 - 1) \stackrel{?}{=} 2 + 2$

$2(3 - 1) \stackrel{?}{=} 4$

$2(2) \stackrel{?}{=} 4$

$4 = 4$ True

▶ Sometimes, equations may have infinitely many solutions or no solutions. When a solution process ends in a *true numerical statement*, the original equation is called an identity, and its solution set is all the real numbers.

Solve: $k + 3 + 7k = 5 + 8k - 2$

$k + 3 + 7k = 5 + 8k - 2$ ← Identify like terms.

$8k + 3 = 3 + 8k$ ← Combine like terms.

$8k - 8k + 3 = 3 + 8k - 8k$ ← Use the Subtraction Property of Equality.

$3 = 3$ ← *true* numerical statement

Think
$k + 3 + 7k = 5 + 8k - 2$ is an identity.

Solution set: $\{k | k \text{ is any real number}\}$.

When a solution process ends in a *false numerical statement*, the original equation is a contradiction, and its solution set is the null set.

Solve: $-3y + 7 + 5y = -11 + 2y$

$-3y + 7 + 5y = -11 + 2y$ ← Identify like terms.

$2y + 7 = -11 + 2y$ ← Combine like terms.

$2y - 2y + 7 = -11 + 2y - 2y$ ← Use the Subtraction Property of Equality.

$7 = -11$ ← *false* numerical statement

Solution set: $\varnothing$

Think
$-3y + 7 + 5y = -11 + 2y$ is a contradiction.

Principles for Solving Equations in One Variable

1. Remove grouping symbols by applying the Distributive Property.
2. Simplify the expressions on each side by combining like terms that are on the same side of the equation.
3. Use the Addition and/or Subtraction Property of Equality to position all of the unknown terms on one side and the numbers without unknown terms on the other side.
4. Simplify the expressions on each side of the equal sign.
5. Use the Multiplication and/or Division Property of Equality to isolate the unknown term and solve the equation.
6. Check the result in the original equation. Work each side separately.
7. State the solution set.
   If the steps above lead to a *true* numerical statement, the solution set is {real numbers}.
   If the steps above lead to a *false* numerical statement, the solution set is $\varnothing$.

Try These

Solve each equation. Check your solution.

1. $\frac{1}{7}(35 + 7x) = 4$ **2.** $12 = 7 - (p + 4)$ **3.** $\frac{1}{2}(4j - 6) + \frac{2}{3}(3j) = 1$ **4.** $6n - 3 = 10n + 17$

5. $7(0.1d - 1) = d + 2$ **6.** $2g + g - 7 = 1 + 3g$ **7.** $t + 1 + 5t = 3 - 2 + t$ **8.** $-5r - 1 + 4r = -1 - r$

Write and solve an equation for each problem.

9. Nick drew a rectangle that is 4 times as long as it is wide. Stacey increased each dimension of Nick's rectangle by 5 inches. If the perimeter of Stacey's rectangle is 60 inches, what are the dimensions of Nick's rectangle?

10. Claire's cousin is now 3 times as old as Claire. In 10 years, her cousin will be twice as old as Claire. How old is Claire now?

11. Derek drew a square and an equilateral triangle that are equal in perimeter. If a side of the equilateral triangle is 1 inch longer than a side of the square, what is the area of the square?

12. **Discuss and Write** Ms. Kinney wrote the equation $4x + 16 = 80$ on the board and asked her class to find the value of $x + 4$. Penny said she would solve the given equation for x and then use that value to find $x + 4$. The class agreed that Penny's method is correct. Explain how you can find the value of $x + 4$ without first solving for x.

Go to PRACTICE BOOK Lesson 2-5 for exercise sets.