Objective To use dimensional analysis to understand, plan, and check the solutions of problems

One way to use **dimensional analysis** is to help plan how to solve a problem.

A garden snail crawls a distance of 38 centimeters in 30 seconds. At what average speed, in inches per hour, did the snail crawl?

▶ Write an equation to find the average speed, r, in terms of the distance, d, and time, t. Then use dimensional analysis to determine which form of the conversion factors are needed.

- Solve for r in the distance formula: $d = rt \longrightarrow \frac{d}{t} = \frac{rt}{t} \longrightarrow \frac{d}{t} = r$

- Write equations that relate the units you need to the units given in the problem: 1 in. = 2.54 cm, 1 h = 3600 s

- Write the original unit for speed as a fraction. Then write the units in the equations above as fractions and multiply so that the original units can be divided out and the desired unit for speed remains.

$$\frac{\cancel{centimeters}}{\cancel{seconds}} \cdot \frac{inches}{\cancel{centimeters}} \cdot \frac{\cancel{seconds}}{hours} = \frac{inches}{hours}$$

- Use the analysis above as a guide to compute the answer.

$$r = \frac{38 \text{ cm}}{30 \text{ s}} \cdot \frac{1 \text{ in.}}{2.54 \text{ cm}} \cdot \frac{3600 \text{ s}}{1 \text{ h}} \approx 1.8 \times 10^3$$

So, the snail crawled at an average speed of about 1.8×10^3 inches per hour.

▶ You can also use dimensional analysis to check that the answer to a real-world problem is reasonable.

When Paula drops a ball from a tree branch, it takes 0.96 second to hit the ground. How high is the branch? Round to the nearest tenth.

- To find how high the branch is, use the formula $h = 0.5gt^2$, where t is time, in seconds, and g is the acceleration due to gravity, $9.81 \frac{m}{s^2}$.

$$h = 0.5gt^2 = (0.5)\left(\frac{9.81 \text{ m}}{\cancel{s^2}}\right)(0.96 \cancel{s})^2 \approx 4.5 \text{ m}$$

- Substitute only the units into the formula to check that your answer has the correct units. $\frac{meters}{\cancel{seconds^2}} \cdot \frac{\cancel{seconds}}{1} \cdot \frac{\cancel{seconds}}{1} = meters$

So, the branch is about 4.5 m above the ground.

Key Concept
Dimensional analysis is the use of units of measurement to guide the solving of problems.

Remember
distance = rate • time
$d = rt$

Remember
$10^3 = 10 • 10 • 10 = 1000$

Use dimensional analysis to convert the units. Round to the nearest tenth.

1. 22 kilometers in 1.5 hours to meters per second _____

2. Discuss and Write Use dimensional analysis and the equation $d = rt$ to explain why the speed of an object cannot be expressed in $\frac{m^2}{s}$.

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State the units for the quantity *y* in the formula from the given information.

3. $y = \dfrac{m}{l^3}$; *m* in g, *l* in cm _____

4. $y = \frac{1}{2}mv^2$; *m* in kg, *v* in m/s _____

5. $y = 4\pi r^2$; *r* in cm _____

6. $y = \dfrac{gm^2}{r^2}$; *g* in $\dfrac{m^3}{kg \bullet s^2}$, *m* in kg, *r* in m _____

Use dimensional analysis to decide whether the answer is reasonable.

7. Hiram runs on a winding path through the woods that is 3.0 km long for a total of 20 min. He calculates that his average speed was 0.15 kilometer per square minute.

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8. Jeff receives a box in the mail that has a length and width of 50 cm and a height of 20 cm. He calculates that the volume of the box is 50,000 cm³.

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9. A sample of a material has a mass of 200 g. Its volume is 20 mL. The material is a solid. Louise calculates that the density of the material is $10\,\dfrac{g}{mL^3}$.

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10. Fran wants to paint all 6 sides of a wooden block. It has the shape of a cube, with each edge measuring 3 cm. Fran computes that the surface area of the cube is 54 cm³.

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Problem Solving

Solve. Use dimensional analysis to plan your approach and check your answer.

11. What is the volume of a rectangular box, in cubic centimeters, with sides of length 0.8 m, 55 cm, and 36 mm?

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12. The speed limit on a certain highway is 65 miles per hour. What is the speed limit expressed in meters per second? Round to the nearest whole number. (Hint: 1 mi ≈ 1609 m)

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13. A painter can paint 200,000 cm² of wall in one hour. At that rate, how many square meters can three painters paint in three hours?

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14. A block in the shape of a cube measures 8 cm on a side. How many blocks can fit in a rectangular box with a length and width of 0.64 m and a height of 0.40 m?

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EXPLAIN YOUR REASONING

15. In physics, the pascal is a unit of pressure. The pressure applied to an object is calculated by dividing the force, measured in $\dfrac{kg \bullet m}{s^2}$, by the area, in m², over which the force is applied. What is another way to write the units for pressure? Explain.

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