

# Formulas and Literal Equations

**Objective** To solve formulas for particular variables • To solve literal equations for particular variables



Finish times are recorded for the runners of a race. If Connor won his 200-meter race in 41.5 seconds, what was his average speed?

You can determine Connor's average speed by using a *formula* that relates distance, rate of speed, and time.

▶ A **formula** is an equation that states a rule for a relationship among particular quantities.

To determine Connor's average speed, use the formula  $d = rt$ , where  $d$  represents distance,  $r$  represents rate of speed, and  $t$  represents time.

$$d = rt$$

$$200 = r \cdot 41.5 \quad \leftarrow \text{Substitute 200 for } d \text{ and 41.5 for } t.$$

$$\frac{200}{41.5} = \frac{r \cdot 41.5}{41.5} \quad \leftarrow \text{Use the Division Property of Equality.}$$

$$4.8 \approx r \quad \leftarrow \text{Round to the nearest tenth.}$$

So Connor's average speed was about 4.8 meters per second.



▶ To *solve for a variable* in a given formula, isolate that variable. Identify the operations on that variable, and use inverse operations to undo them.

If you wanted to know the average speed of the other runners in that race, it would be efficient to first “rearrange” (transform) the formula to isolate the variable  $r$ . This is called *solving for a variable*.

$$d = rt$$

$$\frac{d}{t} = \frac{rt}{t} \quad \rightarrow \quad \frac{d}{t} = r$$

## Example

- 1**  $F = \frac{9}{5}C + 32$  is a formula for temperature in degrees Fahrenheit,  $F$ , in terms of temperature in degrees Celsius,  $C$ . Solve for  $C$ .

$$F = \frac{9}{5}C + 32$$

$$F - 32 = \frac{9}{5}C + 32 - 32 \quad \leftarrow \text{Use the Subtraction Property of Equality.}$$

$$F - 32 = \frac{9}{5}C \quad \leftarrow \text{Simplify.}$$

$$\frac{5}{9}(F - 32) = \frac{5}{9} \cdot \frac{9}{5}C \quad \leftarrow \text{Use the Multiplication Property of Equality.}$$

$$\frac{5}{9}(F - 32) = C \quad \leftarrow \text{solution of the formula for } C$$

- A formula is a type of **literal equation**, which is an equation with two or more variables. In a given literal equation, you can solve for any one of the variables in terms of the others.

## Examples

**1** Solve for  $d$ :  $3(d + m) = 8$

$$3d + 3m = 8 \quad \leftarrow \text{Use the Distributive Property.}$$

$$3d + 3m - 3m = 8 - 3m \quad \leftarrow \text{Use the Subtraction Property of Equality.}$$

$$3d = 8 - 3m \quad \leftarrow \text{Simplify.}$$

$$\frac{3d}{3} = \frac{8 - 3m}{3} \quad \leftarrow \text{Use the Division Property of Equality.}$$

$$d = \frac{8 - 3m}{3} \quad \leftarrow \text{Simplify.}$$

$$\text{or } d = \frac{8}{3} - m$$

**2** Solve for  $z$ :  $12z - a = 3z + 7a$

$$12z - 3z - a = 3z - 3z + 7a \quad \leftarrow \text{Use the Subtraction Property of Equality.}$$

$$9z - a = 7a \quad \leftarrow \text{Combine the like } z\text{-terms.}$$

$$9z - a + a = 7a + a \quad \leftarrow \text{Use the Addition Property of Equality.}$$

$$9z = 8a \quad \leftarrow \text{Combine the like } a\text{-terms.}$$

$$\frac{9z}{9} = \frac{8a}{9} \quad \leftarrow \text{Use the Division Property of Equality.}$$

$$z = \frac{8a}{9} \quad \leftarrow \text{Simplify.}$$

## Try These

Solve for the indicated variable.

1. Solve for  $x$ :

$$y = mx + b$$

2. Solve for  $h$ :

$$V = \frac{1}{3}\pi r^2 h$$

3. Solve for  $x$ :

$$y - y_1 = m(x - x_1)$$

4. Solve for  $y$ :

$$7y + 2h = 3y + 5h$$

5. Solve for  $m$ :

$$q = \frac{m - 10x}{w}$$

6. Solve for  $b_1$ :

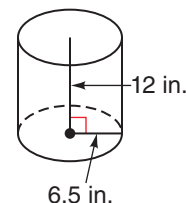
$$A = \frac{1}{2}h(b_1 + b_2)$$

Use the given formula to find the indicated quantity.

7. Shoe sizes and foot length are related by the formula  $S = 3F - 24$ , where  $S$  represents the shoe size and  $F$  represents the length of the foot, in inches. How long is the foot of a person who wears a size  $7\frac{1}{2}$  shoe?

8. The formula  $V = \pi r^2 h$  can be used to find the volume,  $V$ , of a right circular cylinder with radius  $r$  and height  $h$ . What is the volume of the cylinder shown at the right?

9. The formula  $S = 2\pi rh + 2\pi r^2$  can be used to find the surface area,  $S$ , of a right circular cylinder with radius  $r$  and height  $h$ . What is the surface area of the cylinder shown at the right?



10. **Discuss and Write** Albert Einstein's famous formula,  $E = mc^2$ , relates energy,  $E$ , to mass,  $m$ , and the speed of light,  $c$ . Explain how you would solve this formula for  $c$ .