2-9 Problem-Solving Strategy: Solve a Simpler Problem

Objective To solve problems using the strategy *Solve a Simpler Problem*

Problem 1: For two distinct real numbers x and y, where x is not equal to $-y$, is the product $(x - y)(y - x)(x + y)(y + x)$ positive or negative?

Read **Read to understand what is being asked.**

List the facts and restate the question.

Facts: Two *different* real numbers x and y are involved.
x is not equal to $-y$.

Question: Four different numbers are created by adding and subtracting the two given real numbers in different orders. Then the four new numbers are multiplied. Is the product positive or negative?

Plan **Select a strategy.**

When a problem involves relationships among variables, trying out specific values for the variables can be a straightforward way to see what relationships exist.

You can apply the strategy *Solve a Simpler Problem*. It will be easiest to use numbers that make the computation simple, such as two integers.

Problem-Solving Strategies
1. Make a Drawing
2. Solve a Simpler Problem
3. Reason Logically
4. Consider Extreme Cases
5. Work Backward
6. Find a Pattern
7. Account for All Possibilities
8. Adopt a Different Point of View
9. Guess and Test
10. Organize Data

Solve **Apply the strategy.**

Try $x = 5$ and $y = 3$.
$(5 - 3)(3 - 5)(5 + 3)(3 + 5) = (2)(-2)(8)(8)$, which is negative.

Try another pair of numbers with *different characteristics*.

Two negative numbers might be a good choice. Let $x = -2$ and $y = -5$.
$[-2 - (-5)][-5 - (-2)][-2 + (-5)][-5 + (-2)] = (3)(-3)(-7)(-7)$, which is negative.

Notice that no matter what numbers are chosen for x and y, the two differences will just be opposites of each other. Therefore, their product will be negative. The two sums are equal numbers, so their product will be positive. Therefore, the product of the four numbers will be a positive *times* a negative, which will always be negative.

Check **Check to make sure your answer makes sense.**

Are there any cases where the relationship would *not* hold? If x and y were equal or if x and $-y$ were equal, the product would be 0; but the problem excludes these cases.

Would the same answer hold true if x and y are fractions or irrational numbers?

Yes, because the rules for the signs in a product hold for all real numbers, not only for integers.

Problem 2: The sum of the interior-angle measures of all 10-sided polygons is the same. What is this sum?

Read **Read to understand what is being asked.**

List the facts and restate the question.

Facts: The sum of the interior-angle measures is the same for all 10-sided polygons.

Question: What is the sum of the interior-angle measures of a 10-sided polygon?

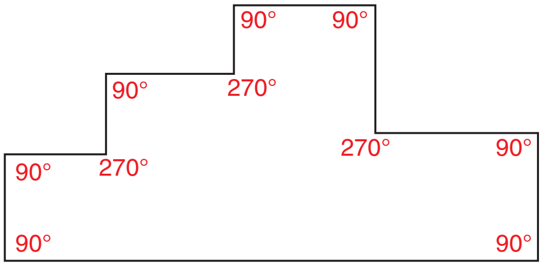
Plan **Select a strategy.**

Because the shape of the polygon does *not* matter, use the strategy *Solve a Simpler Problem*. Draw a polygon for which finding the angle sum is easy.

Solve **Apply the strategy.**

Draw a 10-sided polygon like the one below, in which adjacent sides are perpendicular. Each interior angle is either a right angle or a reflex angle. The measures of each reflex angle is $360° - 90°$, or $270°$. Specifically, there are seven $90°$ angles and three $270°$ angles.

A *reflex angle* is an angle greater than 180° and less than 360°.



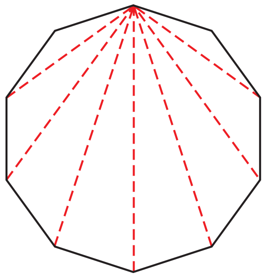
The angle sum is $7(90°) + 3(270°)$, or $1440°$.

So the sum of the angle measures of any 10-sided polygon is $1440°$.

Check **Check to make sure your answer makes sense.**

Solve the problem a different way.

Draw a 10-sided polygon, as shown below. By drawing all the diagonals from one vertex, you can divide the 10-sided polygon into 8 triangles.



Since the sum of the angle measures of a triangle is $180°$, the sum of the angle measures for the polygon is $8(180°)$, or $1440°$. This agrees with the answer above.

Go to PRACTICE BOOK Lesson 2-9 for exercise sets.