

# Write and Graph Inequalities

**Objective** To translate a word sentence into an inequality and vice versa • To identify equivalent symbolic forms for representing the solution sets of inequalities • To connect symbolic and graphic representations of inequalities

The Rockville School wants to donate at least \$1500 to a relief fund. The student council decides to raise the money through a student car wash. The cost for customers will be \$12 per car. What algebraic sentence can be written to represent the number of cars that will be needed in order to raise at least the intended dollar amount?

To represent the number of cars as an algebraic sentence, translate the word sentences into an **inequality**.

- An inequality is a statement that compares two expressions or quantities that may not be equal. It uses one or more **comparison symbols**.

To write an inequality:

- Look for the key words associated with inequality symbols.
- Associate the remaining word phrases with symbols, and use a variable to represent the unknown quantity.

Algebraic Word Sentence	Comparison Symbol
<i>a is less than b.</i>	$a < b$
<i>a is greater than b.</i>	$a > b$
<i>a is less than or equal to b.</i> <i>a is at most b.</i>	$a \leq b$
<i>a is greater than or equal to b.</i> <i>a is at least b.</i>	$a \geq b$
<i>a is not equal to b.</i>	$a \neq b$

cost per car    times    number of cars washed    is at least    intended dollar amount

$\downarrow$                        $\downarrow$                        $\downarrow$                        $\downarrow$                        $\downarrow$   
 12                      •                      c                       $\geq$                       1500

So  $12c \geq 1500$  is an inequality that represents the number of cars that will have to be washed in order to raise at least 1500.

This inequality can also be written as  $1500 \leq 12c$ .

If you reverse the order of the expressions in the inequality, you must *reverse* the inequality symbol.

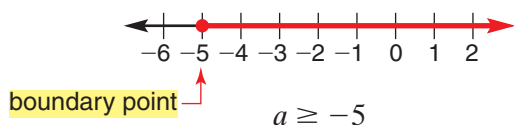
- Since an inequality may have too many solutions to list, you can use a graph on a number line to show all the solutions.

Graph:  $\{a | a \geq -5\}$

**Think.**

Graph all real numbers (not just integers) greater than or equal to  $-5$

To graph  $a \geq -5$ , place a *dot* at  $-5$  on the number line, and then shade the number line to the right of  $-5$ . The dot denotes that  $-5$  is a solution of  $a \geq -5$ .

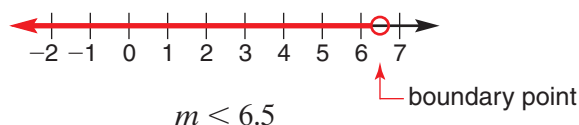


Graph:  $6.5 > m$

**Think.**

$m < 6.5$ ; graph all real numbers less than  $6.5$

To graph  $m < 6.5$ , place a *circle* at  $6.5$  on the number line, and then shade the number line to the left of  $6.5$ . The circle denotes that  $6.5$  is *not* a solution of  $6.5 > m$ .



► **Interval notation** shows the endpoints of a solution set. All the real numbers between these endpoints are in the interval.

**Set-Builder Notation**      **Interval Notation**

$\{x|x > 10\}$       or       $(10, \infty)$

Read both as:  $x$  is greater than 10.  
10 is *not* included in the solution set.



When using interval notation to represent a solution set, the symbol:

$( )$  means *not included* or *open*.



$[ ]$  means *included* or *closed*.

$\infty$  means that the interval continues endlessly in the positive direction.

$-\infty$  means that the interval continues endlessly in the negative direction.

Set-Builder Notation	Interval Notation	Graph
$\{x x \geq -3\}$	Closed at Left, Nonending at Right: $[-3, \infty)$	
$\{x x \leq 1\}$	Nonending at Left, Closed at Right: $(-\infty, 1]$	

► You can also write a solution set to an inequality to represent a given graph and then describe a situation for it.

Graph	Symbolic Notation	Words
	$t \geq -2$ $\{t t \geq -2\}$ $[-2, \infty)$ ← -2 is included in the interval, which continues without end in the positive direction.	The recorded temperature <i>is no less than</i> $-2^{\circ}\text{C}$ .
	$x \leq 175$ $\{x x \leq 175\}$ $(-\infty, 175]$ ← 175 is included in the interval, which continues without end in the negative direction.	The interval includes all real numbers that <i>are no more than</i> 175.

## Try These

Define a variable, and write an inequality for each word sentence.

- Jamie's arcade game score is more than 1200 points.
- Miss Jones said each student's class party expense is to be a maximum of \$15.
- The temperature of Al's mixture in chemistry lab did not exceed  $72^{\circ}\text{F}$ .
- The carpeted area of the school library is less than  $82\text{ ft}^2$ .

Rewrite each solution set in a different symbolic notation.

Then graph each solution set.

- $h < 4$
- $\{x|x > 16\}$
- $w \leq 190.5$
- $(-\infty, \infty)$
- $[80, \infty)$

- Discuss and Write** The Transitive Property of Equality states that if  $a = b$  and  $b = c$ , then  $a = c$ . Do you think there is a Transitive Property of Inequality using the symbol  $>$ ? Give examples to justify your answer. Include negative integers.