3-3 Solve Inequalities Using Multiplication or Division

Objective To solve one-step inequalities using the Multiplication or the Division Properties of Inequality • To graph the solution sets of multiplication and division inequalities

PROBLEM-SOLVING STRATEGY 3

The senior class is going on a field trip. The school buses hold 52 passengers, and there are at least 235 teachers and students going on the trip. What is the minimum number of buses needed?

To find the number of buses the school will need, write and solve a multiplication inequality.

Let b = the number of buses.

number of passengers for one bus	*times*	number of buses	*is at least*	total number of teachers & students
52	•	b	$\geq$	235 ← multiplication inequality

▶ To solve a multiplication inequality, use the Division Property of Inequality.

Solve: $52b \geq 235$

$\frac{52b}{52} \geq \frac{235}{52}$ ← Use the Division Property of Inequality.

$b \geq 4.52$ ← Since the answer represents buses, it will need to be the next greater whole number.

$b \geq 5$

The answer to the problem is 5. However, the solution set to the inequality contains *all the integers* greater than or equal to 5.

The school will need at least 5 buses.

Key Concept

Division Property of Inequality

If a, b, and c are real numbers, c is positive, and $a < b$, then $a \div c < b \div c$.

If a, b, and c are real numbers, c is negative, and $a < b$, then $a \div c > b \div c$.

Similar statements can be written for $a > b$, $a \leq b$, and $a \geq b$.

▶ When you divide both sides of an inequality by a *negative* number, you must *reverse* the inequality symbol to get a true statement.

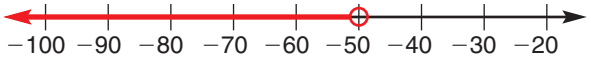
Solve: $-7.30n > 365$

$\frac{-7.30n}{-7.30} < \frac{365}{-7.30}$ ← Divide by a negative number; reverse the inequality symbol.

$n < -50$ ← Simplify.

$\{n | n < -50\}$ or $(-\infty, -50)$ ← solution set

Graph:



Think

The solution set contains *all real numbers* less than −50.

Check: According to the graph, −60 is in the solution set, and −30 is *not*.

Try $n = -60$.

$-7.30n > 365$

$-7.30(-60) \stackrel{?}{>} 365$ ← Substitute −60 for n.

$438 > 365$ True

Remember: Check using the original inequality.

Try $n = -30$.

$-7.30(-30) \stackrel{?}{>} 365$ ← Substitute −30 for n.

$219 > 365$ False

▶ To solve a division inequality, use the Multiplication Property of Inequality.

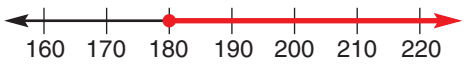
Solve the inequality. Then graph and check the solution set.

Solve: $\frac{x}{12} \geq 15$ ← division inequality

$\frac{x}{12} \cdot 12 \geq 15 \cdot 12$ ← Use the Multiplication Property of Inequality.

$x \geq 180$

Graph: $\{x \mid x \geq 180\}$ or $[180, \infty)$



Check: According to the graph, 204 is in the solution set, and 132 is *not*.

Try $x = 204$.

$\frac{x}{12} \geq 15$

$\frac{204}{12} \stackrel{?}{\geq} 15$ Substitute 204 for x.

$17 \geq 15$ True

Try $x = 132$.

$\frac{x}{12} \geq 15$

$\frac{132}{12} \stackrel{?}{\geq} 15$ Substitute 132 for x.

$11 \geq 15$ False

Key Concept

Multiplication Property of Inequality

If a, b, and c are real numbers, c is positive, and $a < b$, then $ac < bc$.

If a, b, and c are real numbers, c is negative, and $a < b$, then $ac > bc$.

Similar statements can be written for $a > b$, $a \leq b$, and $a \geq b$.

▶ When you multiply both sides of an inequality by a *negative* number, you must *reverse* the inequality symbol to get a true statement.

Solve the inequality. Then graph and check the solution set.

Solve: $-y - \frac{1}{5}y < 30$

$-y + -\frac{1}{5}y < 30$ ← Identify like terms.

$-\frac{6}{5}y < 30$ ← Simplify; combine like terms.

$\left(-\frac{5}{6}\right)\left(-\frac{6}{5}y\right) > 30\left(-\frac{5}{6}\right)$ ← Multiply by a negative number; reverse the inequality symbol.

$y > -25$

Graph: $\{y \mid y > -25\}$ or $(-25, \infty)$

Check: According to the graph, 0 is in the solution set, and -40 is *not*.

$-y - \frac{1}{5}y < 30$

$0 - \frac{1}{5}(0) \stackrel{?}{<} 30$

$0 < 30$ True

$-y - \frac{1}{5}y < 30$

$-(-40) - \frac{1}{5}(-40) \stackrel{?}{<} 30$

$48 < 30$ False

Try These

Solve each inequality. Then graph and check the solution set.

1. $4k < 24$ **2.** $-1 \geq \frac{-x}{11}$ **3.** $-12x + 4x < -4$ **4.** $-20 < -8z - 2z$ **5.** $\frac{3}{5} < \frac{r}{-5}$

6. Monica's new car averages 23 miles per gallon of gasoline. What is the greatest number of gallons of gasoline she will need if she travels no more than 500 miles?

7. **Discuss and Write** Explain, using a number line or model, why it is necessary to reverse the inequality symbol when multiplying and dividing by a negative number.

Go to PRACTICE BOOK Lesson 3-3 for exercise sets.