3-4 Solve Multistep Inequalities

Objective **To solve and graph multistep inequalities**

Sally wants to cater a party, but she needs to spend less than \$800. If the caterer charges \$350 to set up the event and \$12.50 per person, how many guests can Sally invite?

To find all the possible different numbers of guests Sally can invite, write and solve a multistep inequality.

► A multistep inequality involves more than one operation. To solve a multistep inequality, isolate the variable by using the properties of inequality or the inverse of each operation.

Let g = the number of guests.

per person charge *times* number of guests	*plus*	set-up fee	*is less than*	spending limit	
$12.50g$	$+$	350	$<$	800	← multistep inequality

Solve: $12.50g + 350 < 800$

$12.50g + 350 - 350 < 800 - 350$ ← Use the Subtraction Property of Inequality.

$12.50g < 450$

$\frac{12.50g}{12.50} < \frac{450}{12.50}$ ← Use the Division Property of Inequality.

$g < 36$

Graph: $\{g \mid g < 36\}$

Check: According to the graph, 30 is in the solution set, and 40 is *not*.

Try $g = 30$.

$12.50g + 350 < 800$

$12.50(30) + 350 \overset{?}{<} 800$ ← Substitute 30 for g.

$375 + 350 \overset{?}{<} 800$

$725 < 800$ True

Remember: Check by substitution using the original inequality.

Try $g = 40$.

$12.50g + 350 < 800$

$12.50(40) + 350 \overset{?}{<} 800$ ← Substitute 40 for g.

$500 + 350 \overset{?}{<} 800$

$850 < 800$ False

So if the party is catered, Sally will invite fewer than 36 guests.

Example

Solve the inequality. Then graph and check the solution set.

1 **Solve:** $5 + 10 \geq \frac{1}{5}c + 20 - \frac{2}{5}c$

$5 + 10 \geq \frac{1}{5}c + 20 + \left(-\frac{2}{5}c\right)$ ← Identify like terms.

$15 \geq -\frac{1}{5}c + 20$ ← Simplify; combine like terms.

$15 - 20 \geq -\frac{1}{5}c + 20 - 20$ ← Use the Subtraction Property of Inequality.

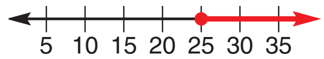
$-5 \geq -\frac{c}{5}$ ← $-\frac{1}{5}c = -\frac{c}{5}$

$-5 \bullet -5 \leq -\frac{c}{5} \bullet -5$ ← Use the Multiplication Property of Inequality.

$25 \leq c$ ← Simplify.

$c \geq 25$

Graph: $\{c \mid c \geq 25\}$ or $[25, \infty)$



Check: According to the graph, 30 is in the solution set.

Try $c = 30$.

$5 + 10 \geq \frac{1}{5}c + 20 - \frac{2}{5}c$

$5 + 10 \overset{?}{\geq} \frac{1}{5}(30) + 20 - \frac{2}{5}(30)$ ← Substitute.

$15 \overset{?}{\geq} 6 + 20 - 12$

$15 \geq 14$ True

► You can use the Distributive Property to help solve inequalities that contain grouping symbols.

Solve: $3(2y + 1) - 2 \leq 31$

$3(2y + 1) - 2 \leq 31$ ← Use the Distributive Property.

$6y + 3 + -2 \leq 31$ ← Identify and combine like terms.

$6y + 1 \leq 31$

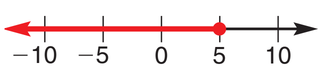
$6y - 1 + 1 \leq 31 - 1$ ← Use the Subtraction Property of Inequality.

$6y \leq 30$

$\frac{6y}{6} \leq \frac{30y}{6}$ ← Use the Division Property of Inequality.

$y \leq 5$

Graph: $\{y \mid y \leq 5\}$ or $(-\infty, 5]$



Check: According to the graph, 0 is in the solution set, and 10 is *not*.

Remember: Follow the Order of Operations.

Try $y = 0$.

$3(2y + 1) - 2 \leq 31$

$3(2 \bullet 0 + 1) - 2 \overset{?}{\leq} 31$ ← Substitute.

$3(0 + 1) - 2 \overset{?}{\leq} 31$

$3(1) - 2 \overset{?}{\leq} 31$

$1 \leq 31$ True

Try $y = 10$.

$3(2y + 1) - 2 \leq 31$

$3(2 \bullet 10 + 1) - 2 \overset{?}{\leq} 31$ ← Substitute.

$3(20 + 1) - 2 \overset{?}{\leq} 31$

$3(21) - 2 \overset{?}{\leq} 31$

$61 \leq 31$ False

Continue Lesson ➡

▶ Sometimes you can also solve a multistep inequality by simplifying its terms using division.

Solve: $4(-2n + 8) > -16$

$\frac{4(-2n + 8)}{4} > \frac{-16}{4}$ ← Since -16 is divisible by 4, divide both sides by 4.

$-2n + 8 > -4$ ← Simplify.

$-2n + 8 - 8 > -4 - 8$ ← Use the Subtraction Property of Inequality.

$-2n > -12$

$\frac{-2n}{-2} < \frac{-12}{-2}$ ← Use the Division Property of Inequality.

$n < 6$

Check: According to the graph, 1 is in the solution set.

Try $n = 1$.

$4(-2n + 8) > -16$

$4(-2 \cdot 1 + 8) \overset{?}{>} -16$ ← Substitute.

$4(-2 + 8) \overset{?}{>} -16$

$4(6) \overset{?}{>} -16$

$24 > -16$ True

Graph: $\{n \mid n < 6\}$ or $(-\infty, 6)$

▶ Sometimes it is easier to work with integers in an inequality than with fractions or decimals. To create an equivalent inequality with coefficients that are integers, *multiply each term* of an inequality with fractions by the LCD. If the inequality includes decimals, *multiply each term* of the inequality by a power of 10.

Solve the inequality. Then graph and check the solution set.

Solve: $-\frac{x}{3} + \frac{1}{4} \le \frac{1}{6}$

$12\left(-\frac{x}{3} + \frac{1}{4}\right) \le 12\left(\frac{1}{6}\right)$ ← The LCD of the fractions is 12. Multiply both sides by 12.

$12\left(-\frac{x}{3}\right) + 12\left(\frac{1}{4}\right) \le 12\left(\frac{1}{6}\right)$ ← Use the Distributive Property.

$-4x + 3 \le 2$

$-4x + 3 - 3 \le 2 - 3$ ← Use the Subtraction Property of Inequality.

$-4x \le -1$

$\frac{-4x}{-4} \ge \frac{-1}{-4}$ ← Use the Division Property of Inequality.

$x \ge \frac{1}{4}$

Check: According to the graph, 3 is in the solution set.

Try $x = 3$.

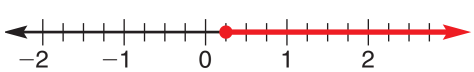
$-\frac{x}{3} + \frac{1}{4} \le \frac{1}{6}$

$-\frac{3}{3} + \frac{1}{4} \overset{?}{\le} \frac{1}{6}$ ← Substitute.

$-1 + \frac{1}{4} \overset{?}{\le} \frac{1}{6}$

$-\frac{3}{4} \le \frac{1}{6}$ True

Graph: $\left\{x \mid x \ge \frac{1}{4}\right\}$ or $\left[\frac{1}{4}, \infty\right)$



Example

1 **Solve:** $-0.3 + 4.4h < 5.2$

$10(-0.3 + 4.4h) < 10(5.2)$ ← Multiply both sides by 10 to create an equivalent inequality with integers.

$10(-0.3) + 10(4.4h) < 10(5.2)$ ← Use the Distributive Property.

$-3 + 44h < 52$ ← Simplify.

$-3 + 3 + 44h < 52 + 3$ ← Use the Addition Property of Inequality.

$44h < 55$ ← Simplify.

$\frac{44h}{44} < \frac{55}{44}$ ← Use the Division Property of Inequality.

$h < \frac{5}{4}$ ← Simplify.

$h < 1.25$ ← $\frac{5}{4} = 1\frac{1}{4} = 1.25$

Graph: $\{h \mid h < 1.25\}$ or $(-\infty, 1.25)$

Check: According to the graph, 0 is in the solution set.

Try $h = 0$.

$-0.3 + 4.4h < 5.2$

$-0.3 + 4.4(0) \overset{?}{<} 5.2$ ← Substitute.

$-0.3 + 0 \overset{?}{<} 5.2$

$-0.3 < 5.2$ True

▶ Solving inequalities with variables on both sides is similar to solving equations with variables on both sides.

PROBLEM-SOLVING STRATEGY 2

Solve: $\frac{3x}{8} - 2 < \frac{5x}{8}$

$\frac{3x}{8} - \frac{3x}{8} - 2 < \frac{5x}{8} - \frac{3x}{8}$ ← Use the Subtraction Property of Inequality.

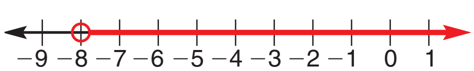
$-2 < \frac{2x}{8}$

$\frac{8}{2} \bullet -2 < \frac{2x}{8} \bullet \frac{8}{2}$ ← Use the Multiplication Property of Inequality.

$-8 < x$

$x > -8$

Graph: $\{x \mid x > -8\}$ or $(-8, \infty)$



Check: According to the graph, 0 is in the solution set.

Try $x = 0$.

$\frac{3x}{8} - 2 < \frac{5x}{8}$

$\frac{3}{8}(0) - 2 \overset{?}{<} \frac{5}{8}(0)$ ← Substitute.

$-2 < 0$ True

Try These

Solve each inequality. Then graph and check the solution set.

1. $9y + 12 < -12$

2. $15 > -2m - 5$

3. $10 \leq -7 - 4h - 2$

4. $2(d + 5) + 8d \geq 4$

5. $3x - (7x - 11) > 9$

6. $\frac{2}{3} + \frac{1}{4}c \geq \frac{3}{4}$

7. $1 - 0.4r \leq 3.4 - 0.6r$

8. $-2k + 8 < \frac{2k}{3}$

9. Discuss and Write Mark solved the inequality $-2x + 7 < 13$ and got the solution $x > -3$, but Jane got $-3 < x$. Who is correct? Explain your response.

Go to PRACTICE BOOK Lesson 3-4 for exercise sets.