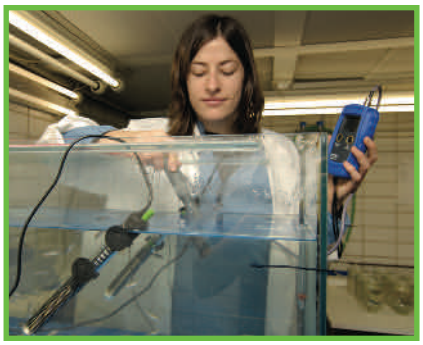
3-5 Solve Compound Inequalities

Objective To write and solve compound inequalities involving AND • To write and solve compound inequalities involving OR

Every day, the caretaker at the university's biology lab checks the water quality in the lab's aquarium. The ideal pH level recommended for freshwater aquariums is between 6.5 and 7.5. How can the caretaker make a graph to show all the possible values of the pH?



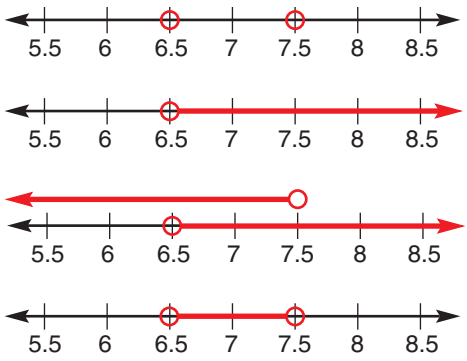
To make a graph that shows all the possible values of the pH, first write the range as a compound inequality or as two simple inequalities joined by the word AND. Then graph the inequality.

► A compound inequality joined by the word AND is called a conjunction. Solutions that satisfy *both* parts of the compound inequality are the solution to the conjunction.

Let p = the pH between 6.5 and 7.5.
Then $p > 6.5$ AND $p < 7.5$. ← compound inequality

► To graph a compound inequality joined by AND:

- Draw and label a number line that includes the boundary points. The boundary points are the points where you start the graph—that is, 6.5 and 7.5.
- Graph the first inequality: $p > 6.5$
- On the same number line, graph the second inequality: $p < 7.5$
- Identify the region where the two graphs overlap. This is the graph of the solution set. The *intersection* satisfies *both* inequalities.



The solution set of the compound inequality is: $\{p \mid 6.5 < p < 7.5\}$.

So the graph represents the pH range between 6.5 and 7.5, not including either endpoint.

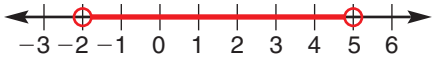
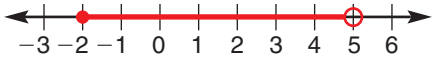
Key Concept

Shortened Form of a Compound Inequality Joined by *And*

If a, b, and c are real numbers, $a < b$ and $b < c$, then $a < b < c$.

This statement is also true if $<$ is replaced by $\leq$, $>$, or $\geq$.

► The table below illustrates some graphs, symbols, and words that represent a compound inequality joined by the conjunction AND.

Graph	Symbolic Notation	Words
	$x > -2$ AND $x < 5$ $\{x \mid -2 < x < 5\}$ ← set-builder notation $(-2, 5)$ ← interval notation	All real numbers greater than −2 AND less than 5
	$x \geq -2$ AND $x < 5$ $\{x \mid -2 \leq x < 5\}$ $[-2, 5)$	All real numbers greater than or equal to −2 AND less than 5

You can solve compound inequalities that are *conjunctions* algebraically and then represent the solution sets graphically or by using symbolic notation.

Solve: $2 \le x + 6 \le 8$

Think
Use the Properties of Inequality.

$2 \le x + 6$ AND $x + 6 \le 8$ ← Rewrite the inequality using AND.

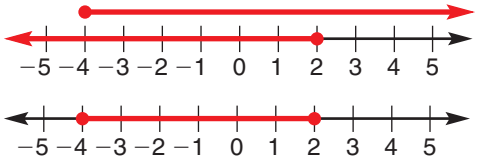
$-6 \quad -6$ $\qquad$ $-6 \quad -6$ ← Solve each simple inequality using the Subtraction Property of Inequality.

$-4 \le x$ AND $x \le 2$

$-4 \le x \le 2$

Graph:

- Graph the first inequality: $x \ge -4$
- Graph the second inequality: $x \le 2$
- Identify the region where the two graphs intersect. This is the graph of the solution set.



Check: Choose a test point to verify the solution set. Try 0.

$2 \le x + 6 \le 8$

$2 \overset{?}{\le} 0 + 6 \overset{?}{\le} 8$ ← Substitute 0 for x.

$2 \le 6 \le 8$ True

Choose a value from the solution set that is *between* −4 and 2, *including both* endpoints. Try 0 as a test point.

So the solution set to the compound inequality is $\{x \mid -4 \le x \le 2\}$. Alternatively, you can write $[-4, 2]$.

Example

1 Solve and check the compound inequality. Graph the solution set.

Solve: $-7 \le 2x - 1 < 3$

$-7 \le 2x - 1$ AND $2x - 1 < 3$ ← Rewrite the inequality using AND.

$+1 \quad +1$ $\qquad$ $+1 \quad +1$ ← Use the Addition Property of Inequality.

$-6 \le 2x$ $\qquad$ $2x < 4$

$\frac{-6}{2} \le \frac{2x}{2}$ $\qquad$ $\frac{2x}{2} < \frac{4}{2}$ ← Use the Division Property of Inequality.

$-3 \le x$ AND $x < 2$

Check: Choose a test point. Try −1 to verify the solution set.

$-7 \le 2x - 1 < 3$

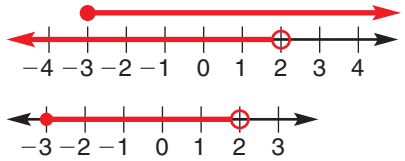
$-7 \overset{?}{\le} 2(-1) - 1 \overset{?}{<} 3$

$-7 \overset{?}{\le} -2 - 1 \overset{?}{<} 3$

$-7 \le -3 < 3$ True

Graph:

- Graph the first inequality: $x \ge -3$
- Graph the second inequality: $x < 2$
- Identify the region where the two graphs intersect. This is the graph of the solution set.



So the solution set to the compound inequality is $\{x \mid -3 \le x < 2\}$. Alternatively, you can write $[-3, 2)$.

Continue Lesson

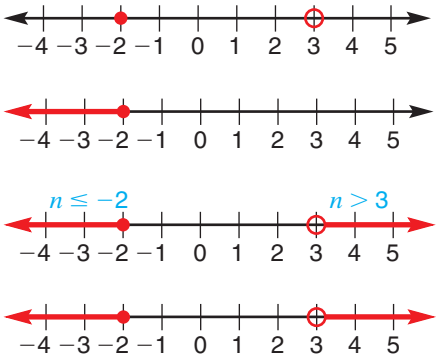
► Two simple inequalities may also be joined by the word OR. This type of compound inequality is called a **disjunction**. Solutions that satisfy at least one part of this type of compound inequality are the solution to the disjunction.

Graph: $n \leq -2$ OR $n > 3$

Graph all numbers that are either *less than or equal to* -2 OR *greater than* 3.

► To graph a compound inequality joined by OR:

- Draw and label a number line that includes the boundary points.
- Graph the first inequality: $n \leq -2$
- On the same number line, graph the second inequality: $n > 3$
- Identify the *union* of the two graphs. This is the graph of the solution set. The union satisfies *both* inequalities.



So the solution set for the graph is $\{n | n \leq -2 \text{ or } n > 3\}$.
Alternatively, you can write $(-\infty, -2] \cup (3, \infty)$.

Remember:
$\cup$ means union.

► You can solve compound inequalities that are *disjunctions* algebraically and then show the solution sets graphically or by using symbolic notation.

Solve and check the compound inequality.
Then graph the solution set.

Solve: $-5 + a > 2$ OR $-5 + a < -3$

Think
Use the Properties of Inequality.

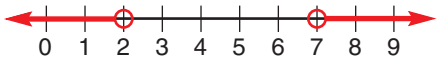
$-5 + a > 2$ OR $-5 + a < -3$

$+5 \quad +5 \qquad +5 \quad +5$ ← Solve each simple inequality using the Addition Property of Inequality.

$a > 7$ OR $a < 2$

Graph:

- The graph identifies the solutions of the compound inequality.
- Identify the union of the two graphs. This is the graph of the solution set.



Check: Choose test points to verify the solution set.

$-5 + a > 2$ OR $-5 + a < -3$

Think
Choose a value from the solution set, which is $a > 7$ OR $a < 2$.

Try 8. Try 1.

$-5 + 8 \stackrel{?}{>} 2$ $\quad$ $-5 + 1 \stackrel{?}{<} -3$

$3 > 2$ True $\quad$ $-4 < -3$ True

So the solution set to the compound inequality is $\{a | a < 2 \text{ or } a > 7\}$.
Alternatively, you can write $(-\infty, 2) \cup (7, \infty)$.

Example

1 Solve and check the compound inequality. Graph the solution set.

Solve: $-4c \geq 8$ OR $6c > 24$

$-4c \geq 8$ OR $6c > 24$

$\frac{-4c}{-4} \leq \frac{8}{-4}$ $\quad$ $\frac{6c}{6} > \frac{24}{6}$ ← Use the Division Property of Inequality.

$c \leq -2$ OR $c > 4$

Check: Choose test points. Try -3 and 6 to verify the solution set.

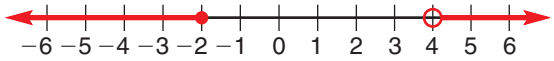
$-4c \geq 8$ OR $6c > 24$

$-4(-3) \stackrel{?}{\geq} 8$ $\quad$ $6(6) \stackrel{?}{>} 24$

$12 \geq 8$ True $\quad$ $36 > 24$ True

Graph:

- The graph identifies the solutions of the compound inequality.
- Identify the union of the two graphs. This is the graph of the solution set.



So the solution set to the compound inequality is $\{c \mid c \leq -2 \text{ or } c > 4\}$.
Alternatively, you can write $(-\infty, -2] \cup (4, \infty)$.

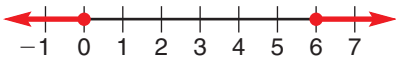
▶ You can write a compound inequality from a graph.

- First, identify the boundary points of the graph. Determine whether they are *included* or *excluded* in the solution set.
- Then use the inequality symbol that corresponds to the direction of the arrow.

Write a compound inequality shown by each graph.

The shaded region on the graph includes -3 and is *between* the values -3 and 4, not including 4. Thus, the compound inequality is a *conjunction*, which involves AND.

So the inequality is $\{x \mid -3 \leq x < 4\}$.
Alternatively, you can write interval notation: $[-3, 4)$.



The shaded regions on the graph are *not between* two values. Thus, the compound inequality is a *disjunction*, which involves OR.

So the inequality is $\{x \mid x \leq 0 \text{ or } x \geq 6\}$.
Alternatively, you can write interval notation: $(-\infty, 0] \cup [6, \infty)$.

Try These

Solve and check each compound inequality. Then graph the solutions.

1. $-4 < x + 1 < 6$ $\quad$ **2.** $-1 \leq -3z + 2 \leq 5$ $\quad$ **3.** $4r - 1 < 7$ AND $4r + 8 > 8$

4. $y + 3 > 13$ OR $y + 3 < -4$ $\quad$ **5.** $-4m + 6 < 18$ OR $-4m + 6 > 42$ $\quad$ **6.** $\frac{3x}{4} < 9$ AND $4x + 8 > 8$

7. Discuss and Write What are the similarities and differences for the graphs of the compound inequalities $x > -1$ OR $x \leq -4$ and $-4 < x \leq -1$?

Go to PRACTICE BOOK Lesson 3-5 for exercise sets.