3-6 Solve Absolute-Value Inequalities

Objective To solve inequalities involving absolute-value expressions

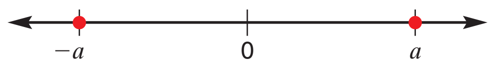
> **Remember:** The absolute value of any real number, x, is the distance from zero to x on a number line.

► Inequalities that contain an absolute-value expression can be written as a compound inequality.

When $a > 0$:

$|x| = a \longrightarrow$ means that x is a units from 0.

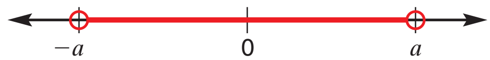
$x = a$ OR $x = -a$



$|x| < a \longrightarrow$ means that x is less than a units from 0.

$x > -a$ AND $x < a$

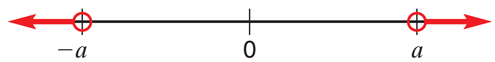
The above is also true for $\leq$.



$|x| > a \longrightarrow$ means that x is greater than a units from 0.

$x < -a$ OR $x > a$

The above is also true for $\geq$.



► To solve an absolute-value inequality involving the "is less than" symbol, rewrite the inequality as a conjunction, and solve.

Key Concept

Principles for Solving an Absolute-Value Inequality

1. Isolate the absolute-value expression.
2. Write the statement as a compound inequality.
3. Solve the two simple inequalities.
4. Graph the solution set and check.

PROBLEM-SOLVING STRATEGY 3

Solve: $|x| + 4 < 14$ ← Isolate the absolute-value expression.

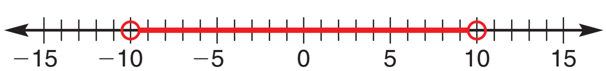
$|x| + 4 - 4 < 14 - 4$ ← Use the Subtraction Property of Inequality.

$|x| < 10$

Rewrite this statement as a *conjunction*.

$x > -10$ AND $x < 10$ ← x is *between* −10 and 10.

Graph: $\{x \mid -10 < x < 10\}$; also written as $(-10, 10)$



Check: Try $x = -5 \longrightarrow$ $|-5| + 4 \overset{?}{<} 14$

$5 + 4 \overset{?}{<} 14$

$9 < 14$ True

Example

1 **Solve:** $|r + 7| - 2.5 \leq 11$

$|r + 7| - 2.5 \leq 11$

$|r + 7| - 2.5 + 2.5 \leq 11 + 2.5$

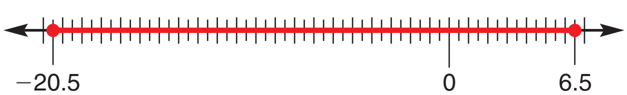
$|r + 7| \leq 13.5$

$r + 7 \geq -13.5$ AND $r + 7 \leq 13.5$

$\underline{\quad -7 \quad -7 \quad}$ $\qquad$ $\underline{\quad -7 \quad -7 \quad}$

$r \geq -20.05$ AND $r \leq 6.5$

Graph: $\{r \mid -20.5 \leq r \leq 6.5\}$; also written as $[-20.5, 6.5]$



Check: Try $r = 0. \longrightarrow$ $|0 + 7| - 2.5 \overset{?}{\leq} 11$

$|7| - 2.5 \overset{?}{\leq} 11$

$4.5 \leq 11$ True

▶ To solve an absolute-value inequality involving the "is greater than" symbol, rewrite the inequality as a disjunction, and solve.

Solve: $|x| - 3 > 5$ ← Isolate the absolute-value expression.

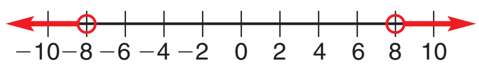
$|x| - 3 + 3 > 5 + 3$ ← Use the Addition Property of Inequality.

$|x| > 8$

Rewrite this statement as a *disjunction*.

$x < -8$ OR $x > 8$ ← x is either less than −8 OR greater than 8.

Graph: $\{x \mid x < -8 \text{ or } x > 8\}$; also written as $(-\infty, -8) \cup (8, \infty)$



Check: Try $x = -10$ → $|-10| - 3 \overset{?}{>} 5$

$10 - 3 \overset{?}{>} 5$

$7 > 5$ True

Example

1 **Solve:** $2|c - 1| + 14 \geq 26$ ← Isolate the absolute-value expression.

$2|c - 1| + 14 - 14 \geq 26 - 14$ ← Use the Subtraction Property of Inequality.

$\frac{2|c - 1|}{2} \geq \frac{12}{2}$ ← Use the Division Property of Inequality.

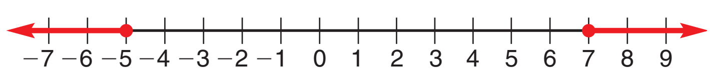
$|c - 1| \geq 6$ ← Rewrite this statement as a *disjunction*.

$c - 1 \leq -6$ OR $c - 1 \geq 6$

$\quad +1 \quad +1 \qquad +1 \quad +1$ ← Use the Addition Property of Inequality.

$c \leq -5$ OR $c \geq 7$ ← c is either less than or equal to −5 OR greater than or equal to 7.

Graph: $\{c \mid c \leq -5 \text{ OR } c \geq 7\}$; also written as $(-\infty, -5]$ OR $[7, \infty)$



Check: Try $r = -8$.

$2|-8 - 1| + 14 \overset{?}{\geq} 26$

$2|-9| + 14 \overset{?}{\geq} 26$

$2(9) + 14 \overset{?}{\geq} 26$

$32 \geq 26$ True

Try These

Solve each inequality. Then graph and check the solution set.

1. $|m| - 16 < 4$

2. $16|d| \geq 4$

3. $|x + 11| - 4 \leq 9$

4. $\frac{4}{5}|d| > 3$

5. $|y + 6| - 2\frac{1}{4} \geq 1\frac{3}{4}$

6. $|2x| - 1 < 11$

7. $-2 + |3b - 5| > 8$

8. $2.4 + |g - 1| \geq 1.5$

9. **Discuss and Write** Explain the similarities and differences between solving an inequality that contains an absolute-value expression and one that does not. Use specific examples to support your reasoning.

Go to PRACTICE BOOK Lesson 3-6 for exercise sets.