3-8 Problem-Solving Strategy: Reason Logically

Objective To solve problems using the strategy *Reason Logically*

Problem 1: Is there a two-digit whole number with the property that, when the digits are reversed, the resulting number is twice the original number?

Read **Read to understand what is being asked.**

List the facts and restate the question.

Facts: The digits of a two-digit number can be reversed to form another two-digit number.

Question: Is there a two-digit number for which the number that results from reversing the digits is twice the original number?

Plan **Select a strategy.**

You could examine all two-digit numbers. However, that could take some time. It might be easier to use the strategy *Reason Logically*.

Problem-Solving Strategies

1. Make a Drawing
2. Solve a Simpler Problem
3. Reason Logically
4. Consider Extreme Cases
5. Work Backward
6. Find a Pattern
7. Account for All Possibilities
8. Adopt a Different Point of View
9. Guess and Test
10. Organize Data

Solve **Apply the strategy.**

- Since the "reversed" number must be *double* the original, then it must be *a multiple of 2*. Its units digit must be 0, 2, 4, 6, or 8. But it cannot be 0 because that would mean the tens digit of the original number is 0, which is not possible for a two-digit number. So the original number has to have a tens digit of 2, 4, 6, or 8.
- Since the "reversed" number is *twice* the original, then the units digit of the original number must be *twice* the tens digit. The only numbers that satisfy both these criteria are 24 and 48. Neither of these numbers has a "reverse" that is twice the original.

Therefore, there is no two-digit whole number with the property that, when the digits are reversed, the resulting number is twice the original number.

Check **Check to make sure your answer makes sense.**

You can also look at this problem algebraically. Suppose such a number exists. If t is the tens digit and u is the units digit, then the value of the number is $(10t + u)$. If you reverse the digits so u becomes the tens digit and t becomes the units digit, the value of the new number is $(10u + t)$.

If the new number is twice the original, then $(10u + t) = 2(10t + u)$. Solve for t in terms of u, as shown to the right.

$(10u + t) = 2(10t + u)$

$10u + t = 20t + 2u$ ← Apply the Distributive Property.

$8u + t = 20t$ ← Subtract $2u$ from both sides.

$8u = 19t$ ← Subtract t from both sides.

$\frac{8}{19}u = t$ ← Divide both sides by 19.

Note that the tens digit, t, of the original number t is $\frac{8}{19}$ *times* the units digit u. But there is no digit u between 0 and 9, for which $\frac{8}{19}u$ is a digit between 1 and 9. Therefore, there is no two-digit number for which you can reverse the digits to get a new number twice the original.

Problem 2: If you follow the steps given in the example in the table below, the result will be divisible by 9 no matter what number you start with. Explain why this is true.

Steps	Example
1. Start with a four-digit whole number.	4783
2. Multiply the number by 3.	3(4783) = 14,349
3. Add each digit of the result to the result.	14,349 + 1 + 4 + 3 + 4 + 9 = 14,370
4. Multiply the new number by 3.	3(14,370) = 43,110
5. Add each digit of the result to the result.	43,110 + 4 + 3 + 1 + 1 + 0 = 43,119

Read **Read to understand what is being asked.**

List the facts and restate the question.

Facts: You begin with any four-digit number. You triple the number, add each digit of the result to the result, triple the new number, and then add each digit of the result to the result.

Question: Why is the final number divisible by 9 no matter what the initial number is?

Plan **Select a strategy.**

Try using the strategy *Reason Logically*, using some basic divisibility rules.

Solve **Apply the strategy.**

- Let n represent the original number in Step 1.
- In Step 2, when you triple the number, n, the result is $3n$. Because $3n$ is divisible by 3, you know that the *sum of its digits*, s, is divisible by 3. Then $s = 3m$, where m is a whole number.
- In Step 3, when you add the sum of the digits of $3m$ to $3n$, the result is $3n + 3m$, or $3(n + m)$.
- In Step 4, when you triple the number, $3(n + m)$, the result is $3 \bullet 3(n + m)$, or $9(n + m)$.

  Let $k = n + m$. Since n and m are whole numbers, then k is a whole number and $9(n + m) = 9k$. Since $9k$ is divisible by 9, the *sum of its digits*, S, must be divisible by 9. Then $S = 9M$, where M is a whole number.
- In Step 5, the result is $9k + 9M$, or $9(k + M)$, which is divisible by 9.

Remember:
A number is divisible by 3 *if and only if* the sum of its digits is divisible by 3.

A number is divisible by 9 *if and only if* the sum of its digits is divisible by 9.

Check **Check to make sure your answer makes sense.**

Read back through the solution. Make sure each step follows logically from the one before. Start with a couple of different four-digit numbers and use a handheld to verify that the steps in the solution are true for that specific number. For example:

- Start with 5156 and triple it. The result:15,468. Verify that it is divisible by 3.
- Add each digit of 15,468 to 15,468. (15,468 + 1 + 5 + 4 + 6 + 8 = 15,492). Verify that 15,492 is divisible by 3.
- Multiply 15,492 by 3. The result is 46,476. Verify that 46,476, is divisible by 9.
- Add each digit of 46,476 to 46,476. Verify that the result is divisible by 9.

Go to PRACTICE BOOK Lesson 3-8 for exercise sets.