4-2 Introduction to Functions

Objective To define a function • To identify relations as functions, given different representations • To apply the vertical-line test to graphs • To use function notation

PROBLEM-SOLVING STRATEGY 1

Mike is planning to plant vegetables in a rectangular section of his garden. To protect the vegetables from foraging deer, Mike intends to fence in the section. If the length of the vegetable garden is to be 40 feet, how much fencing will Mike need?

To find the amount of fencing needed, you can use the formula for the perimeter of a rectangle, $P = 2\ell + 2w$.

► The formula for the perimeter of a rectangle gives the relationship between the perimeter and the dimensions of the rectangle.

Since the length of the rectangular section is given, the perimeter will depend upon the width he chooses.

$P = 2\ell + 2w \rightarrow P = 2(40) + 2w$ ← Substitute the given length into the Perimeter Formula.

$P = 80 + 2w$

To record some possibilities for the perimeter relation, use a table.

- The rule for this relation is $P = 80 + 2w$.
- The ordered pairs are of the form (w, P).
- The domain for this relation, the possible values for w, is limited to positive real numbers.
- The range for this relation, the possible values for P, is also limited to positive real numbers.

w	$80 + 2w$	P	(w, P)
10	80 + 2(10)	100	(10, 100)
25	80 + 2(25)	130	(25, 130)
37.5	80 + 2(37.5)	155	(37.5, 155)
40	80 + 2(40)	160	(40, 160)

So Mike might need 100 ft, 130 ft, 155 ft, or 160 ft of fencing depending on the width of the garden.

In this relation, when you input a positive real number for the value of w, there is *exactly one output value* for P. Such a relation is called a function.

Key Concept

Function

A *function* is a special type of relation that pairs each domain value with exactly one range value.

Example

Write the domain and range for the relation and tell whether the relation is a function.

1

x	y
−2	16
−1	1
0	0
1	1
2	16

Think
The table represents a set of ordered pairs (x, y).

Domain = $\{-2, -1, 0, 1, 2\}$

Range = $\{0, 1, 16\}$

The set of ordered pairs in this table does represent a function because every domain value is paired with exactly one range value.

▶ To determine if a set of ordered pairs is a function, check if no first element can be paired with more than one second element. It does not matter if some second elements are repeated.

Determine whether each relation is a function. Explain.

• (0, 7), (1, 7), (2, 9), (3, 16) ← No domain value has more than one range value.

These ordered pairs define a function.

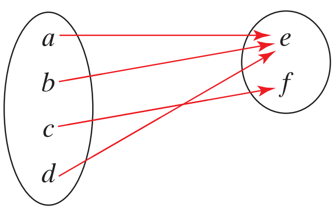
• (0, 7), (0, 9), (2, 9), (3, 16) ← The domain value 0 has more than one range value.

These ordered pairs *do not* define a function.

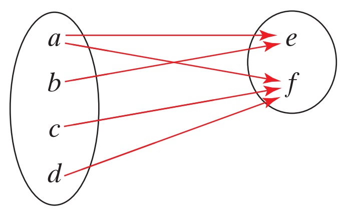
Remember:
The *domain* of a relation is the set of all *first* elements of the ordered pairs.
The *range* of a relation is the set of all *second* elements of the ordered pairs.

▶ To determine if a relation represented by a mapping diagram is a function, use the arrows to find the pairing for each domain value.

Determine whether each relation is a function. Explain.



There is only one arrow leading away from each domain value. So each domain value is paired with exactly one range value. This mapping diagram represents a relation that is a function.

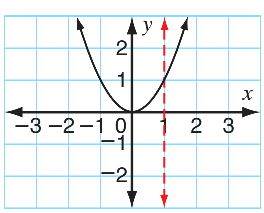


There is more than one arrow leading away from domain value *a*. So not every domain value is paired with exactly one range value. This mapping diagram represents a relation that is not a function.

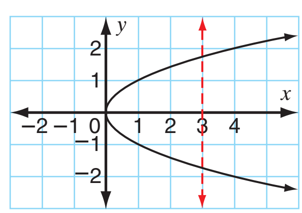
▶ To determine if a relation represented by a graph is a function, use the vertical-line test. When any vertical line intersects a graph:

• at exactly one point, the graph represents a function.
• at more than one point, the graph does not represent function.

Which of these graphs represent a function? Explain your answer.



Each vertical line drawn through the graph intersects the graph at exactly one point. This graph represents a relation that is a function.



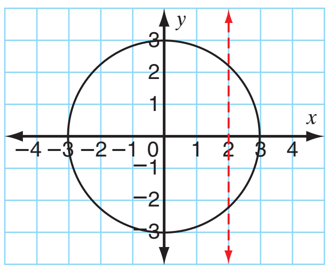
A vertical line drawn through the graph intersects the graph at more than one point. This graph represents a relation that is not a function.

Continue Lesson ➡

Examples

Determine whether each represents a function. Explain.

1



Think Use the vertical-line test on the graph.

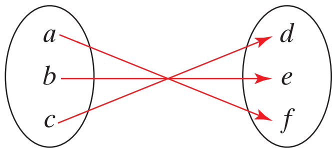
$\text{Domain} = \{x \mid -3 \le x \le 3\}$

$\text{Range} = \{y \mid -3 \le y \le 3\}$

There are infinitely many ordered pairs on this graph.

This graph represents a relation that is not a function because at least one vertical line will intersect the graph at more than one point.

2



Think Is there more than one arrow going out from any domain value?

Ordered Pairs: (a, f), (b, e), (c, d)

$\text{Domain} = \{a, b, c\}$

$\text{Range} = \{d, e, f\}$

This mapping diagram represents a relation that is a function because each domain value is paired with exactly one range value.

► A function rule is an equation that describes a function. Sometimes the equation is written using function notation.

To write a rule in function notation, you use the symbol $f(x)$ in place of y.

You read $f(x)$ as "f of x."

Function notation allows you to see the input value, x. The table at the right shows the names or symbols used with a function.

domain	range
input	output
x	$f(x)$
x	y
independent variable	dependent variable

Function Rule: $y = 4x$ ← This equation describes a relationship between y and x. It allows you to find ordered pairs that satisfy this relation.

Function Notation: $f(x) = 4x$ ← The symbol $f(x)$ (read "f of x") replaces y. It shows that x represents the *input values*.

$f(3) = 4(3)$ ← The symbol $f(3)$ (read "f of 3") specifies an input value for x.

$= 12$ ← the corresponding *output value*

So the ordered pair (3, 12) satisfies the function rule $f(x) = 4x$. Inputting other values for the variable x will determine other ordered pairs that satisfy this function rule.

► To distinguish among different functions of the same variable, different letters are used in the function notation. For example, if x is the independent variable, you may see $f(x)$, $g(x)$, and $h(x)$.

PROBLEM-SOLVING STRATEGY 3

If $f(x) = 5x + 7$ and $g(x) = x^2 - 12$, find $f(-4) + g(3)$.

• Substitute -4 for x in $f(x)$.

$f(x) = 5x + 7$

$f(-4) = 5(-4) + 7$

$= -20 + 7 = -13$

• Substitute 3 for x in $g(x)$.

$g(x) = x^2 - 12$

$g(3) = 3^2 - 12$

$= 9 - 12 = -3$

• Determine the required sum.

$f(-4) + g(3)$

$-13 + (-3)$

-16

Examples

Evaluate each function. Write the ordered pair to show the correspondence.

1 $f(x) = 3x - 4$, for $f(-4)$

$f(-4) = 3(-4) - 4$

$= -12 - 4 = -16$

So the ordered pair is $(-4, -16)$.

2 $f(x) = -x^2$, for $f(6)$

$f(6) = -6^2$

$= -36$

So the ordered pair is $(6, -36)$.

3 If $f(x) = 3x + 6$ and $h(x) = (x + 1)^2$, find $2[f(-3) + h(1.5)]$.

• Substitute -3 for x in $f(x)$.

$f(x) = 3x + 6$

$f(-3) = 3(-3) + 6$

$= -9 + 6 = -3$

• Substitute 1.5 for x in $h(x)$.

$h(x) = (x - 1)^2$

$h(1.5) = (1.5 - 1)^2$

$= (0.5)^2 = 0.25$

• Determine the required product.

$2[f(-3) + h(1.5)]$

$2[-3 + 0.25]$

$2[-2.75]$

-5.5

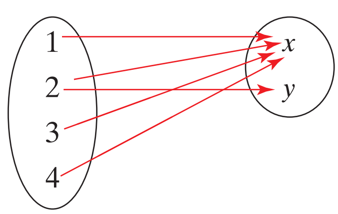
Try These

Write the domain and range for the relation and tell whether the relation is a function.

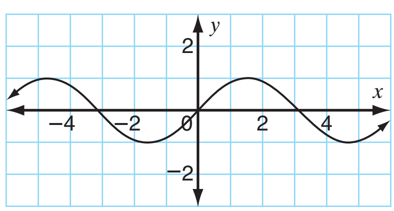
1.

x	y
−2	2
−1	1
0	0
1	1
2	2

2.



3.



Evaluate the expression, given $f(x) = 3x + 4$, $g(x) = x^2 - 4x + 1$, and $h(x) = \sqrt{x}$.

4. $f(-2) + g(5)$

5. $f(1.5) + g(-2) - h(9)$

6. $2[f(4) + g(8)]$

7. $\dfrac{f(5) + g(0)}{h(25)}$

8. Discuss and Write Hal said that the relation $\{(-8, 16), (-6, 16), (-6, 12)\}$ is not a function because 16 appears twice in the ordered pairs. Do you agree with Hal's reasoning? Is this relation a function? Explain.

Go to PRACTICE BOOK Lesson 4-2 for exercise sets.