4-4 Arithmetic Sequences

Objective To recognize and extend arithmetic sequences • To find an indicated term of an arithmetic sequence • To write a function rule for an arithmetic sequence

Marisol is an accomplished skydiver. Suppose in today's freefall jump, Marisol fell 16 feet in the first second, 48 feet in the next second, 80 feet in the third second, and 112 feet in the fourth second. If Marisol continued to fall at this rate, how many feet would she fall in the fifth second?

To find how many feet she would fall in the fifth second, look for a pattern that occurred during the first four seconds.

► To look for a pattern, organize the data by using a table or by using a graph.

• Use a Table

Time (seconds)	1	2	3	4	5
Distance (feet)	16	48	80	112	?

+ 32 + 32 + 32 ← Add 32 to get the next term in the pattern.

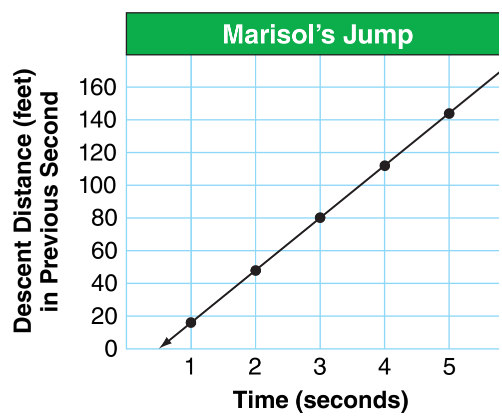
According to the pattern, you can find the number of feet Marisol fell in the fifth second by adding 32 to the number of feet she fell in the fourth second. So in the fifth second, she fell 112 + 32, or 144, feet.

• Use a Graph

When the information about Marisol's jump is shown on a graph, the points appear to fall on a straight line.

Applying the vertical-line test to this graph shows that this relation is a function.

You can find the number of feet Marisol fell during the fifth second and sixth second by continuing the pattern: 16, 48, 80, 112, $112 + 32 = 144$, $144 + 32 = 176$, …



Remember: An ellipsis (three dots) indicates that a pattern continues.

► An ordered set of elements that follows a pattern is called a sequence. A sequence, such as 16, 48, 80, 112, which has a first term and a last term, is called a finite sequence. A sequence such as 16, 48, 80, 112, . . . , in which there is another term after each term of the sequence, is called an infinite sequence.

In the sequence whose terms are $a_1, a_2, a_3, \ldots, a_{n-1}, a_n \ldots$, a_1 represents the first term, a_3 represents the third term, and a_n represents the *n*th term of the sequence. The term before the *n*th term is represented by a_{n-1}. The subscript in a term represents the place of the term in the sequence.

A sequence in which each term after the first is found by *adding* a nonzero constant (called the common difference, d) to the previous term is called an arithmetic sequence.

The sequence 16, 48, 80, 112, … is an arithmetic sequence, and the constant difference is 32.

Examples

Determine if each sequence is an arithmetic sequence. If the sequence is arithmetic, find the next term.

1 2, 3, 5, 8, …

Think Test for a common difference.

2nd term − 1st term = 3 − 2 = 1

3rd term − 2nd term = 5 − 3 = 2

The differences are not the same.

So 2, 3, 5, 8, … is *not* an arithmetic sequence.

2 42, 19, −4, −27, …

2nd term − 1st term = 19 − 42 = −23

3rd term − 2nd term = −4 − 19 = −23

4th term − 3rd term = −27 − (−4) = −23

There is a common difference, $d = -23$.

So 42, 19, −4, −27, … is an arithmetic sequence. The next term is −27 + (−23), or −50.

Each term of an arithmetic sequence can be related to the first term, a_1, and the common difference, d. The sequence may be written as follows:

Number of Term	1	2	3	4	5	…	a_n
	↕	↕	↕	↕	↕		↕
Term of Sequence	a_1	$a_1 + d$	$a_1 + 2d$	$a_1 + 3d$	$a_1 + 4d$	…	$a_1 + (n - 1)d$

In general, the nth term of an arithmetic sequence is the sum of the first term, a_1, and $(n - 1)$ common differences, that is, $a_n = a_1 + (n - 1)d$.

Key Concept

Formula for the nth Term of an Arithmetic Sequence

$a_n = a_1 + (n - 1)d$ where a_n is the nth term, a_1 is the first term, n is the number of terms, and d ($d \neq 0$) is the common difference.

Find a_{10} of −0.2, −0.8, −1.4, −2, …

- Find d by subtracting the first term from the second term.

$d = a_2 - a_1 = -0.8 - (-0.2) = -0.6$

- Use the formula to find the nth term of an arithmetic sequence.

$a_n = a_1 + (n - 1)d$

$a_{10} = -0.2 + (10 - 1)(-0.6)$ ← Substitute 10 for n, −0.2 for a_1, and −0.6 for d.

$= -0.2 + 9(-0.6) = -0.2 + (-5.4)$ ← Simplify.

$= -5.6$

Examples

1 Find a_{16} of $1, \frac{2}{3}, \frac{1}{3}, 0, \ldots$

$d = a_2 - a_1 = \frac{2}{3} - 1 = -\frac{1}{3}$ ← Find d by subtracting the first term from the second term.

$a_{16} = 1 + (16 - 1)\left(-\frac{1}{3}\right)$ ← Substitute 16 for n, 1 for a_1, and $-\frac{1}{3}$ for d in $a_n = a_1 + (n - 1)d$.

$= 1 + 15\left(-\frac{1}{3}\right) = 1 + (-5)$ ← Simplify.

$= -4$

Continue Lesson ➡

2 The odometer on a motorcycle read 50,500 at the end of a day. Every day thereafter, the motorcycle is driven 35 miles. What is the odometer reading 40 days later?

Since the odometer reading will *increase* by 35 miles per day, this situation can be represented using an arithmetic sequence with $a_1 = 50{,}500$, $d = 35$, and the odometer reading 40 days later as a_{41}.

$a_n = a_1 + (n - 1)d$ ← Write the rule to find the nth term.

$a_{41} = 50{,}500 + (41 - 1)(35)$ ← Substitute the given values.

$= 50{,}500 + (40)(35) = 50{,}500 + 1400$ ← Simplify.

$= 51{,}900$

So the motorcycle's odometer will read 51,900 miles 40 days later.

By writing an algebraic expression for the nth term of a sequence, you are writing a function rule for the sequence.

Write a function rule for the nth term of the arithmetic sequence: 2, 5, 8, 11, …

Think
$d = a_2 - a_1 = 5 - 2 = 3$

Make a table to help identify a relationship between the number of the term, n, and the value of the term, a_n, of the sequence.

Term Number, n	Value of Term, a_n	$a_n = a_1 + (n - 1)d$
1	2	$a_1 = 2$
2	5	$a_2 = 2 + (2 - 1)3 = 2 + 1 \bullet 3 = 5$
3	8	$a_3 = 2 + (3 - 1)3 = 2 + 2 \bullet 3 = 8$
4	11	$a_4 = 2 + (4 - 1)3 = 2 + 3 \bullet 3 = 11$
⋮	⋮	⋮
n	a_n	$a_n = 2 + (n - 1)3$ $= 2 + 3n - 3$ ← Simplify. $= 3n - 1$

Each term, after the first, is 2 *plus* 3 *times* 1 *less than* the term number, n.

Function rule: $a_n = 3n - 1$

Example

1 Write a function rule for the nth of the arithmetic sequence: $-50, -25, 0, 25, 50, \ldots$

$d = a_2 - a_1 = -25 - (-50) = 25$ ← Find d by subtracting the first term from the second term.

$a_n = -50 + (n - 1)(25)$ ← Substitute -50 for a_1, and 25 for d in $a_n = a_1 + (n - 1)d$.

$= -50 + 25n - 25$ ← Simplify.

$= 25n - 75$

Function rule: $a_n = 25n - 75$

Try These

Determine if each sequence is an arithmetic sequence. Use a pattern to write the next four terms.

1. $2, -5, -12, -19, \ldots$
2. $1, 2, 4, 7, \ldots$
3. $t + 8, 3t + 5, 5t + 2, 7t - 1, \ldots$
4. $0.1, 0.01, 0.001, 0.0001, \ldots$
5. $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \ldots$
6. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

Find the indicated term of each arithmetic sequence.

7. a_{13} for $7, 4, 1, -2, \ldots$
8. a_{20} for $-19, -15, -11, -7, \ldots$
9. a_{100} for $0.25, 0.5, 0.75, 1, \ldots$

Write a function rule for the *n*th term of each arithmetic sequence.

10. $2, 4, 6, 8, \ldots$
11. $4, 7, 10, 13, \ldots$
12. $3, 8, 13, 18, \ldots$

13. Mr. and Mrs. Tomkins started a college fund for their son Marcus when he was in the first grade. They began the fund with \$2500, and each year, they increased their contribution to the fund by \$500. What was their contribution to the fund when Marcus was a senior in high school?

14. **Discuss and Write** Josie has written an arithmetic sequence. In her sequence, the third term is 12 and the ninth term is -12. What is the first term of Josie's sequence? Explain your reasoning.

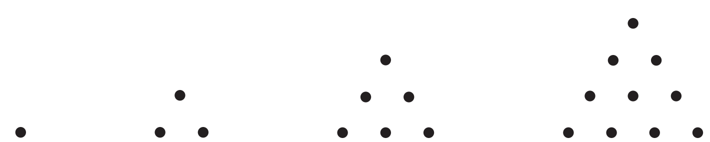
Go to PRACTICE BOOK Lesson 4-4 for exercise sets.

Acceleration: Use Arithmetic Series to Represent Triangular Numbers

Numbers that can be represented by dots arranged in the shapes of certain geometric figures are named for the shapes they form.

The following dot patterns are representations of the first four *triangular numbers*.

1st **2nd** **3rd** **4th**



1, 2 | 1, 2, 3 | 1, 2, 3, 4 ← arithmetic sequence

1 | $1 + 2 = 3$ | $1 + 2 + 3 = 6$ | $1 + 2 + 3 + 4 = 10$ ← arithmetic series

Notice that each triangular number after the first can be written as the *sum* of the terms of a finite arithmetic sequence, called an arithmetic series.

Follow the pattern above. What is the fifth triangular number? 15

What is the tenth triangular number? 55
Explain how you arrived at your answer.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10 ← arithmetic sequence

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$ ← arithmetic series