Name _____

5-10B Inverse Functions

Objective To write an expression for the inverse of a function

Meg and Jim play a number game. First, Meg asks Jim to think of a number. Then Jim follows the game directions at the right to report a number. Jim reports 26. Meg says that he started with 8. How did Meg find Jim's starting number?

Directions for Number Game
1. Add 7 to the number.
2. Multiply the result by 6.
3. Subtract 12 from the product.
4. Divide the difference by 3.
5. Report the resulting quotient.

▶ To find how Meg found Jim's starting number, first write a function that describes how to transform the initial input (the number Jim thinks of), x, into the output (the number Jim reports), y. Then find the **inverse function**.

- To write an equation, represent the directions in order.
  Let x = the number Jim chooses and y = the number Jim reports.

$y = [(x + 7) \bullet 6 - 12] \div 3$ ← Write the steps as an equation.
$y = [6x + 42 - 12] \div 3$ ← Use the Distributive Property.
$y = 6x + 30 \div 3$ ← Subtract like terms.
$y = 2x + 10$ ← Simplify.

- Find the inverse function.

$x = 2y + 10$ ← Switch x and y in the equation.
$x - 10 = 2y + 10 - 10$ ← Use the Subtraction Property of Equality.
$x - 10 = 2y$ ← Simplify.
$(x - 10) \div 2 = 2y \div 2$ ← Use the Division Property of Equality.
$y = 0.5x - 5$ ← Simplify and reverse the sides of the equation.

Key Concept
The inverse of a function of the form $y = f(x)$ is found by switching x and solving for y.

So, Meg takes half of the number Jim reports, then subtracts 5 to find the original number.

Check: $0.5(26) - 5 = 13 - 5 = 8$

Examples

1 Find the inverse function of $y = |x - 2|$.
Two x-values correspond to each y-value greater than 0. For example, both $x = 1$ and $x = 3$ result in $y = 1$. To make sure that the inverse is a function, use a **restricted domain** so that each input has exactly one output. In this case, there is only one output for every input when $x \geq 2$.

A restricted domain is often used to specify a *one-to-one* section of a function.

$y = x - 2, x \geq 2$ ← Use the restricted domain.
$x = y - 2, y \geq 2$ ← Switch x and y.
$y = x + 2, y \geq 2$ ← Solve for y.

The inverse function is $y = x + 2$, but it is only valid for $y \geq 2$.

2 Find the inverse of $y = -0.4x - 2$.

$x = -0.4y - 2$ ← Switch x and y in the equation.
$x + 2 = -0.4y - 2 + 2$ ← Use the Addition Property of Equality.
$x + 2 = -0.4y$ ← Simplify.
$(x + 2) \div (-0.4) = (-0.4y) \div (-0.4)$ ← Use the Division Property of Equality.
$y = -2.5x - 5$ ← Simplify and reverse the sides of the equation.

1. Discuss and Write The ordered pair (4, 18) is a solution to the equation $y = 3x + 6$. Without solving, name an ordered pair that is a solution to the inverse function. Explain how you know it is a solution.

Name _____

5-10B Inverse Functions

Find the inverse of the function.

2. $y = 3(x - 2) + 7$

3. $y = |x + 3|$

4. $y = x + 18$

5. $y = \frac{1}{4}x$

6. $y = 4x - 11$

7. $y = -2x - 8$

8. $y = -|x|$

9. $y = |x| - 1$

Problem Solving

Write equations to answer the questions below.

10. Kyra sells bracelets at her mom's garage sale. Her initial expenses were $5.00 for advertising supplies and $1.50 to make each bracelet. Write an equation for the total cost as a function of the number of bracelets. Then write a function for the number of bracelets Kyra sells as a function of the total cost of the bracelets.

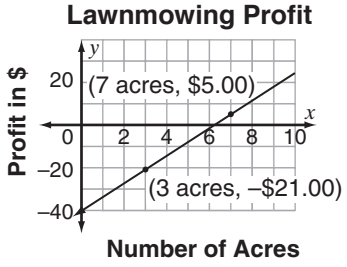
11. To convert temperature between degrees Celsius and degrees Fahrenheit, multiply the Celsius temperature by 1.8 and add 32. Write an equation for the temperature in °F as a function of the temperature in °C. Then write an equation for the temperature in °C as a function of the temperature in °F.

12. Darryl fills a cylindrical pool using a garden hose. The height of the water increases at a constant rate. The table below gives information on the height of the water in the pool at a given time.

Time (minutes)	Water height (inches)
1	3.1
5	3.9

Write an equation for water height as a function of time. Then write an equation for time as a function of water height. Use the second equation to find when the water is 4.12 inches high.

13. Otis estimates the profit for his lawnmowing business will make. He draws a graph of profit as a function of number of acres mowed.



Write an equation for profit as a function of acres mowed. Find how many acres Otis must mow to break even, or have a profit of $0.

CRITICAL THINKING

14. Which two linear functions are the same as their inverses? Why? (Hint: Consider the slope and intercepts of the graph of a function that would be the same as its inverse.)

Use after Online Common Core Lesson 5-10A.