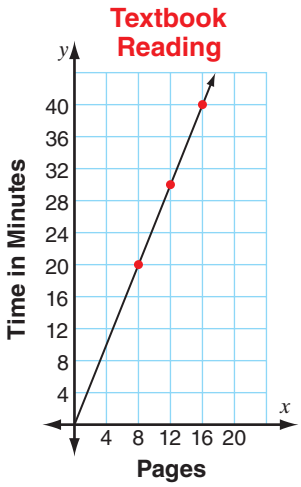
5-2 Direct Variation

Objective To identify and interpret direct variation • To write and graph direct variation

The table below and the graph at the right show how long it took Pablo to read each of the first three chapters of his history book. Is there a mathematical relationship between the number of pages in a chapter and the time it took Pablo to read them?

Chapter	Pages (x)	Time in Minutes (y)
One	8	20
Two	16	40
Three	12	30



As the number of pages in a chapter increases, the amount of time it takes Pablo to read the chapter changes by a constant factor. The time, y, varies *directly* with the number of pages in the chapter, x. This relationship is called a direct variation.

► A direct variation is a linear function that can be written in the form $y = kx$, where k is a nonzero constant called the constant of variation.

constant of variation: $k = \frac{y}{x} = \frac{20}{8} = \frac{40}{16} = \frac{30}{12} = \frac{5}{2}$

Substituting $\frac{5}{2}$ for k in the equation $y = kx$ gives $y = \frac{5}{2}x$.

So the relationship between the number of pages, x, and the time it took Pablo to read them, y, is $y = \frac{5}{2}x$.

► For every direct variation:

- The graph is a straight line that passes through the origin.
- The constant of variation is the slope of the line.

Not *all* linear equations represent direct variations. If a linear equation cannot be written in the form of $y = kx$, then the function is not a direct variation.

Key Concept

Direct Variation

A direct variation is a linear function in which for each ordered pair, (x, y), $y = kx$ or $k = \frac{y}{x}$, $k \neq 0$.

Examples

Tell whether each relation represents a direct variation. If so, give the constant of variation.

1

x	y
−3	6
−2	4
1	−2

Method 1 Find the constant of variation, $\frac{y}{x}$, for each ordered pair.

$k = \frac{y}{x} = \frac{6}{-3} = \frac{4}{-2} = \frac{-2}{1} = -2$

Yes, the ordered pairs represent a direct variation because $\frac{y}{x} = -2$ for each ordered pair. y varies directly with x.

Method 2 Write an equation.

$y = -2x$ ← Each value for y is equal to −2 times the corresponding value for x.

Yes, the ordered pairs represent a direct variation because they can be written in the form $y = kx$, where $k = -2$.

2 $3y = 2x + 6$ ← Solve the equation for y.

$\frac{3y}{3} = \frac{2x + 6}{3}$ ← Use the Division Property of Equality.

$y = \frac{2}{3}x + 2$

The equation is not a direct variation because it cannot be written in the form $y = kx$.

3 $y - x = 0$ ← Solve the equation for y.

$y - x + x = 0 + x$ ← Use the Addition Property of Equality.

$y = x$

The equation is a direct variation because it can be written in the form $y = kx$. The constant of variation is 1.

► You can write and solve direct variation equations.

The value of y varies directly with x, and $y = 9$ when $x = 6$. Find x when $y = 6$.

First find k. Then write an equation and solve for x.

$k = \frac{y}{x} = \frac{9}{6} = \frac{3}{2}$ ← Find the constant of variation.

$y = \frac{3}{2}x$ ← Write the equation, substituting $\frac{3}{2}$ for k.

$6 = \frac{3}{2}x$ ← Substitute 6 for y.

$\frac{2}{3}(6) = \frac{2}{3}\left(\frac{3}{2}x\right)$ ← Use the Multiplication Property of Equality.

$4 = x$

Try These

Explain whether each relation represents a direct variation. If so, state the constant of variation.

1.

x	y
1	−2
−2	−4
4	8

2.

x	y
4	6
6	9
10	15

3. $2y + x = 0$

4. $y - 2 = -3x$

Solve.

5. The value of y varies directly with x, and $y = 7.2$ when $x = 1.6$. Find y when $x = 2.4$.

6. The amount of dog food Bowser eats varies directly with the number of days he is fed. If he eats 0.75 pounds in 2 days, how long will it take him to finish a 30-pound bag? (*Hint:* Let x = number of days and y = amount of food.)

7. Discuss and Write Every direct variation is a linear function, but not every linear function is a direct variation. Explain.

Go to PRACTICE BOOK Lesson 5-2 for exercise sets.