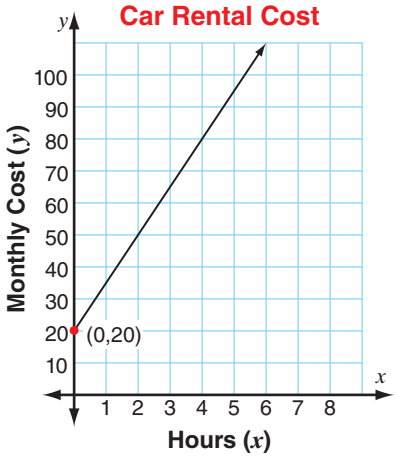
5-3 Equations in Slope-Intercept Form

Objective To graph a line using slope-intercept form • To write a linear equation in slope-intercept form

Mei lives in the city and does not own a car. She pays a $20 monthly fee to Urban Car Rental plus $15 per hour each time she uses one of its cars. You can write an equation to represent Mei's monthly cost.



Let x = the number of hours Mei rents a car.
Let y = total monthly cost.

Total monthly cost = cost *per* hour + monthly fee

$$y = 15 \cdot x + 20$$

The monthly cost increases $15 for every 1 hour Mei rents a car.

The constant rate, or *slope*, of this line is $\frac{15}{1}$, or 15.

Notice that the line crosses, or intersects, the y-axis at the point $(0, 20)$. The y-coordinate of the point where the graph crosses the y-axis is the y-intercept. The y-intercept of this line is 20.

The equation that describes this relationship, $y = 15x + 20$, is in slope-intercept form.When an equation is written in slope-intercept form, $y = mx + b$, the slope, m, is the coefficient of x, and the y-intercept, b, is the constant.

Key Concept

Slope-Intercept Form of a Linear Equation

The slope-intercept form of a linear equation is

$$y = mx + b,$$

where m is the *slope* of the line and b is the y-intercept.

▶ The slope-intercept form, $y = mx + b$, allows you to quickly identify the slope and y-intercept and to graph the line.

Given the equation $y = -\frac{1}{2}x - 1$, identify the slope and y-intercept. Then graph the line.

1 Identify and plot the y-intercept.

$y = -\frac{1}{2}x - 1$

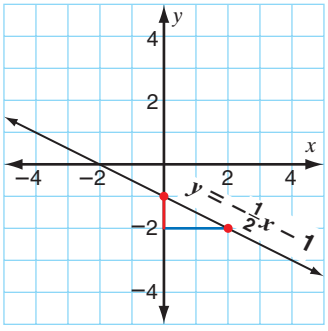
Think
The x-coordinate for the y-intercept is always 0.

y intercept is -1, which is the point $(0, -1)$.

2 Use the slope to plot another point on the line.

$$\text{slope} = \frac{-1}{2} = \frac{\text{vertical change}}{\text{horizontal change}}$$

From $(0, -1)$, count 1 unit down and 2 units to the right to plot another point. Plot the point $(2, -2)$.



3 Draw the line through the two points.

Examples

For each equation, identify the slope and y-intercept, and graph the line.

1 $y = \frac{5}{3}x$

$y = \frac{5}{3}x + 0$

Slope $(m) = \frac{5}{3}$, and

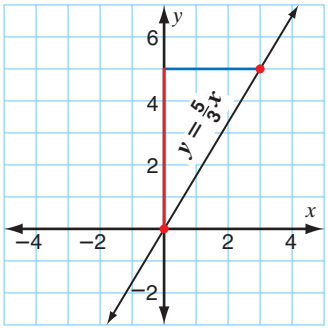
y-intercept $(b) = 0$, so the line contains $(0, 0)$.

$m = \frac{5}{3}; b = 0$

- Plot the y-intercept $(0, 0)$.
- Use the slope to plot another point.

  slope $= \frac{\text{rise}}{\text{run}} = \frac{5}{3}$

  From $(0, 0)$, count 5 units up and 3 units to the right to plot another point. Plot the point $(3, 5)$.
- Draw the line through the two points.



2 $y = -5$

$y = 0 + (-5)$

$y = -5$

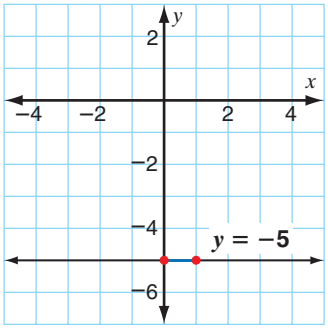
Slope $(m) = 0$, and y-intercept $(b) = -5$, so the line contains $(0, -5)$.

$m = 0; b = -5$

- Plot the y-intercept $(0, -5)$.
- Use the slope to plot another point.

  slope $= 0 = \frac{\text{rise}}{\text{run}} = \frac{0}{1}$

  From $(0, -5)$, count 0 units up and 1 unit to the right to plot another point. Plot the point $(1, -5)$.
- Draw the line through the two points.



▶ Linear equations are *not always* given in slope-intercept form. In this case to find the slope and y-intercept, rewrite the equation by solving for y.

PROBLEM-SOLVING STRATEGY 2

Identify the slope and y-intercept of the line $2y + 2x = 5$. Then graph the line.

Write the equation in the form $y = mx + b$ by solving for y.

$2y + 2x = 5$

$2y + 2x - 2x = -2x + 5$ ← Use the Subtraction Property of Equality.

$2y = -2x + 5$

$\frac{2y}{2} = \frac{-2x + 5}{2}$ ← Use the Division Property of Equality.

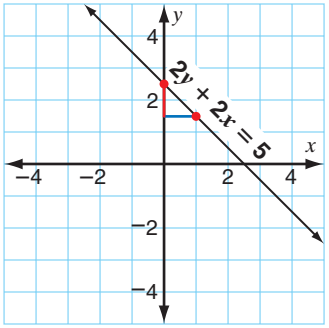
$y = -1x + \frac{5}{2}$

$m = -1; b = \frac{5}{2}$

- Plot the y-intercept, $\left(0, \frac{5}{2}\right)$.
- Use the slope to plot another point.

  Slope $= \frac{\text{rise}}{\text{run}} = \frac{-1}{1} = -1$

  From $\left(0, \frac{5}{2}\right)$, count 1 unit down and 1 unit to the right to plot another point. Plot the point $\left(1, 1\frac{1}{2}\right)$.
- Draw a line through the two points.



Continue Lesson ➡

▶ Given the slope and y-intercept, you can write an equation of a line.

Write an equation of the line in slope-intercept form for the line with slope -2 and y-intercept -5.

$y = mx + b$ ← Use the slope-intercept form.

$y = -2x + (-5)$ ← Substitute the given values in the slope-intercept form.

$y = -2x - 5$ ← Simplify.

The equation is $y = -2x - 5$.

Examples

Write an equation of the line in slope-intercept form.

1 slope $= 0$; y-intercept $= -3$

$y = mx + b$ ← Use the slope-intercept form.

$y = 0(x) + (-3)$ ← Substitute the given values.

$y = -3$ ← Simplify.

So the equation is $y = -3$.

2 slope $= -\frac{8}{5}$; y-intercept $= 12$

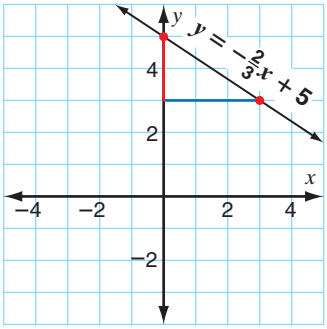
$y = mx + b$ ← Use the slope-intercept form.

$y = -\frac{8}{5}x + 12$ ← Substitute the given values.

So the equation is $y = -\frac{8}{5}x + 12$.

▶ You can write an equation of a line in slope-intercept form, given the graph of a line.

Write the slope-intercept form of the equation of the line shown.



$y = mx + b$ ← Use the slope-intercept form.

$b = 5$ ← The line crosses the y-axis at $y = 5$.

$m = \frac{-2}{3}$ ← The line moves down 2 units for every 3 units it moves to the right.

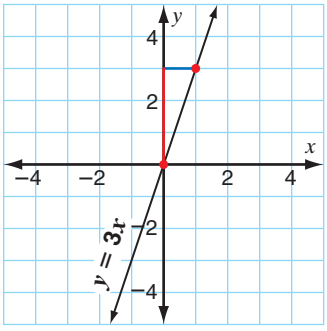
$y = -\frac{2}{3}x + 5$ ← Substitute 5 for b and $\frac{-2}{3}$ for m.

So the equation of the line is $y = -\frac{2}{3}x + 5$.

Example

Write the slope-intercept form of the equation of the line shown.

1



$y = mx + b$ ← Use the slope-intercept form.

$b = 0$ ← The line crosses the y-axis at $y = 0$.

$m = \frac{3}{1}$ ← The line moves up 3 units for every 1 unit it moves to the right.

$y = 3x + 0$ ← Substitute 3 for m and 0 for b.

$y = 3x$ ← Simplify.

So the equation of the line is $y = 3x$.

▶ You can also write an equation of a line in slope-intercept form, given the slope *and* any point on the line.

Write an equation of the line in slope-intercept form with slope $-\frac{5}{4}$ that contains $(-8, 0)$.

Method 1 Use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{-5}{4} = \frac{y - 0}{x - (-8)} \leftarrow \text{Substitute the given values.}$$

$$-5(x + 8) = 4(y) \leftarrow \text{Simplify, and apply the Cross-Products Rule.}$$

$$-5x - 40 = 4y \leftarrow \text{Apply the Distributive Property.}$$

$$\frac{-5x - 40}{4} = \frac{4y}{4} \leftarrow \text{Use the Division Property of Equality.}$$

$$\frac{-5x}{4} - \frac{40}{4} = y$$

$$y = -\frac{5}{4}x - 10 \leftarrow \text{Simplify.}$$

Method 2 Use the slope-intercept form.

$$y = mx + b$$

$$0 = -\frac{5}{4}(-8) + b \leftarrow \text{Substitute the given values.}$$

$$0 = 10 + b \leftarrow \text{Simplify.}$$

$$-10 = b \leftarrow \text{Use the Subtraction Property of Equality to solve for } b.$$

$$y = -\frac{5}{4}x - 10 \leftarrow \text{Substitute } -\frac{5}{4} \text{ for } m \text{ and } -10 \text{ for } b \text{ in } y = mx + b.$$

Try These

Identify the slope and *y*-intercept of the line whose equation is given. Graph the line.

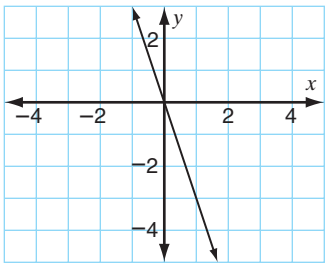
1. $y = x + 7$ **2.** $3y = 8x - 6$ **3.** $2x - 3y = 5$

Write an equation of the line in slope-intercept form with the given slope and *y*-intercept.

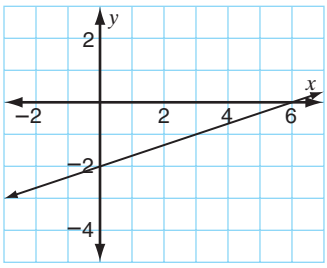
4. slope: $\frac{6}{5}$; *y*-intercept: -8 **5.** slope: -0.8; *y*-intercept: $\frac{4}{7}$ **6.** slope: 0; *y*-intercept: 3.8

Write an equation of each line in slope-intercept form.

7.



8.



Write an equation of the line in slope-intercept form with the given slope and containing the given point.

9. $m = -2$; $(-3.5, 7)$ **10.** $m = \frac{3}{4}$; $(6, -2)$

11. Discuss and Write Can two different lines have the same *y*-intercept? Explain. Provide examples to support your answer.

Go to PRACTICE BOOK Lesson 5-3 for exercise sets.