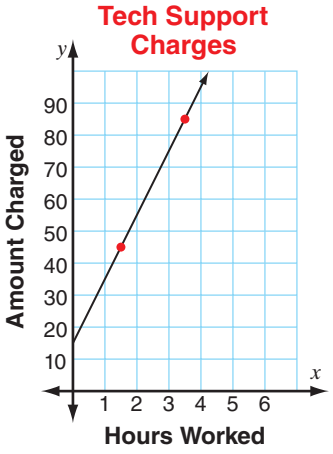
5-4 Equations in Point-Slope Form

Objective To write and graph linear equations using point-slope form

Zach operates a tech support business. He charged one customer \$45 for $1\frac{1}{2}$ hours of work and a second customer \$85.00 for $3\frac{1}{2}$ hours of work. The graph shows the relationship between Zach's fee (y) and how long he works (x). What equation describes this graph?



Find the slope of the line using the points $\left(1\frac{1}{2}, 45\right)$ and $\left(3\frac{1}{2}, 85\right)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{85 - 45}{3\frac{1}{2} - 1\frac{1}{2}} = \frac{40}{2} = 20$$

Suppose (x, y) is another point on the line. The slope of the line through $\left(1\frac{1}{2}, 45\right)$ and (x, y) must also be 20.

Therefore, $\frac{y - 45}{x - 1\frac{1}{2}} = \frac{20}{1}$. ← Use the slope formula.

$y - 45 = 20\left(x - 1\frac{1}{2}\right)$ ← Apply the Cross-Products Rule.

So the equation $y - 45 = 20\left(x - 1\frac{1}{2}\right)$ describes the graph.

This form of a linear equation is called the point-slope form: $y - y_1 = m(x - x_1)$.

Key Concept

Point-Slope Form of a Linear Equation

The equation of a nonvertical line with slope m and through point (x_1, y_1) is $y - y_1 = m(x - x_1)$.

▶ When you know the slope of a line and a point that the line contains, use the point-slope form to write an equation of a line.

Write the point-slope form of a line with slope -2 that passes through $(2, -1.5)$. Then graph the line.

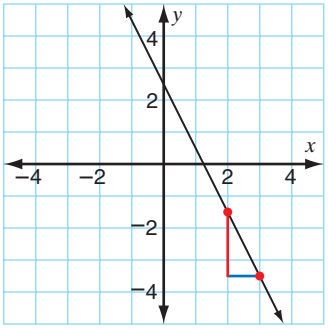
$y - y_1 = m(x - x_1)$ ← Use the point-slope form.

$y - (-1.5) = -2(x - 2)$ ← Substitute the given values.

$y + 1.5 = -2(x - 2)$ ← Simplify.

The point-slope form of the equation is $y + 1.5 = -2(x - 2)$.

- Plot: $(2, -1.5)$
- Use the slope to locate another point on the line.
- Draw the line.



Example

Write the point-slope form of a line with the given slope that contains the given point. Then graph the line.

1 slope: $\frac{2}{3}$; point: $(-2, -1)$

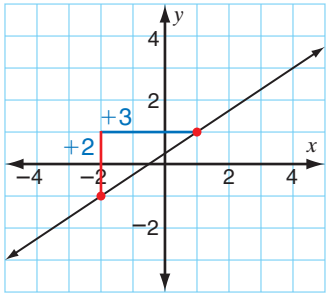
$y - y_1 = m(x - x_1)$ ← Use the point-slope form.

$y - (-1) = \frac{2}{3}[x - (-2)]$ ← Substitute the given values.

$y + 1 = \frac{2}{3}(x + 2)$ ← Simplify.

The point-slope form of the equation is $y + 1 = \frac{2}{3}(x + 2)$.

- Plot: $(-2, -1)$
- Use the slope to locate another point on the line.
- Draw the line.



▶ You can also use the point-slope form to write the equation of a line when you know two points on the line.

- Find the slope using both points.
- Substitute the slope and the coordinates of one of the points into the formula $y - y_1 = m(x - x_1)$.
- Simplify if necessary.

Write an equation of a line in point-slope form passing through the points.

$(-1, 5)$ and $(3, -5)$.

$m = \frac{-5 - 5}{3 - (-1)} = \frac{-10}{4} = \frac{-5}{2}$ ← Find the slope.

$y - y_1 = m(x - x_1)$ ← Use the point-slope form.

Let $(-1, 5) = (x_1, y_1)$ ← Choose one of the points.

$y - 5 = -\frac{5}{2}[x - (-1)]$ ← Substitute 5 for y_1, $-\frac{5}{2}$ for m, and -1 for x_1.

$y - 5 = -\frac{5}{2}(x + 1)$ ← Simplify.

So an equation of the line in point-slope form is $y - 5 = -\frac{5}{2}(x + 1)$.

Example

1 Given points $(5, -9)$ and $(-7, -9)$, write an equation of a line in point-slope form.

$m = \frac{-9 - (-9)}{-7 - 5} = \frac{0}{-12} = 0$ ← Find the slope.

$y - y_1 = m(x - x_1)$ ← Use the point-slope form.

Let $(5, -9) = (x_1, y_1)$ ← Choose one of the points.

$y - (-9) = 0(x - 5)$ ← Substitute -9 for y_1, 0 for m, and 5 for x_1.

$y + 9 = 0(x - 5)$ ← Simplify.

$y + 9 = 0$

So an equation of the line in point-slope form is $y + 9 = 0$.

Try These

Write an equation of a line in point-slope form. Then graph the line.

1. slope: $-\frac{3}{5}$; point: $(0, -6)$

2. slope: 6; point: $(-2.5, 0.5)$

3. $(-3, 0)$ and $(0, -3)$

4. $(-1, 5)$ and $(3, -1)$

5. Discuss and Write Explain how to write an equation of a line in point-slope form when given two points on the line.

Go to PRACTICE BOOK Lesson 5-4 for exercise sets.