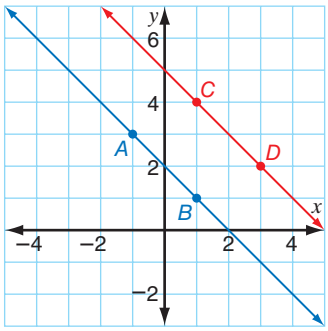
# 5-6 Parallel and Perpendicular Lines

**Objective** To determine if two lines are parallel or perpendicular using slope
- To write an equation of a line that is parallel or perpendicular to a given line

The lines shown in the graph(s) at the right are parallel. They are in the same plane but have no point in common. Notice that they have the same slope but cross the $y$-axis at different points.




The equation of $\overleftrightarrow{AB}$ is $y = -1x + 2$.

The equation of $\overleftrightarrow{CD}$ is $y = -1x + 5$.

**Remember:**
$\overleftrightarrow{AB}$ represents the line that contains points $A$ and $B$.

Lines $AB$ and $CD$ are called a family of graphs because they have at least one characteristic in common. They both have the same slope.

▶ To determine whether lines are parallel:

- Write each equation in slope-intercept form.
- Compare the slopes of the lines. If the slopes are the same and the $y$-intercepts are different, the lines are parallel.

**Key Concept**

**Parallel Lines**
- If two different nonvertical lines have the same slope, then they are parallel.
- If two different nonvertical lines are parallel, then they have the same slope.
- Any two different vertical lines are parallel.

Are the graphs of $y = -\frac{1}{2}x + 1$ and $x + 2y = 6$ parallel? Explain.

Write $x + 2y = 6$ in slope-intercept form.

Compare it with $y = -\frac{1}{2}x + 1$.

$$x - x + 2y = -x + 6 \quad \leftarrow \text{Use the Subtraction Property of Equality.}$$

$$\frac{2y}{2} = \frac{-x + 6}{2} \quad \leftarrow \text{Use the Division Property of Equality.}$$

$$y = -\frac{1}{2}x + 3$$

The slope of $y = -\frac{1}{2}x + 3$ is $-\frac{1}{2}$. The slope of $y = -\frac{1}{2}x + 1$ is $-\frac{1}{2}$.

The lines are parallel. They have the same slope and different $y$-intercepts.

## Examples

**1** Determine whether the lines are parallel.

The equation of $\overleftrightarrow{EF}$ is $6x - 2y = 4$. The equation of $\overleftrightarrow{GH}$ is $y = -3x + 8$.

Write $\overleftrightarrow{EF}$ in slope-intercept form. Compare it with $\overleftrightarrow{GH}$.

$$6x - 6x - 2y = -6x + 4 \quad \leftarrow \text{Use the Subtraction Property of Equality.}$$

$$\frac{-2y}{-2} = \frac{-6x + 4}{-2} \quad \leftarrow \text{Use the Division Property of Equality.}$$

$$y = 3x - 2$$

So the slope of $\overleftrightarrow{EF}$ is 3 and the slope of $\overleftrightarrow{GH}$ is $-3$.

$\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$ are not parallel lines because they do not have the same slope.

**2** Determine whether the lines are parallel.

Given $J(-4, -2)$, $K(-4, 1)$, $L(2, -1)$, and $M(2, 5)$, is $\overleftrightarrow{JK} \parallel \overleftrightarrow{LM}$?

**Remember:**
$\parallel$ is the symbol for "is parallel to."

Find the slope of $\overleftrightarrow{JK}$. ← $J$ is $(-4, -2)$; $K$ is $(-4, 1)$.

$\frac{1-(-2)}{-4-(-4)} = \frac{3}{0}$ ← Substitute the coordinates into the slope formula, and simplify.

The slope of $\overleftrightarrow{JK}$ is *undefined*.

$\overleftrightarrow{JK}$ is a vertical line.

Find the slope of $\overleftrightarrow{LM}$. ← $L$ is $(2, -1)$; $M$ is $(2, 5)$.

$\frac{5-(-1)}{2-2} = \frac{6}{0}$ ← Substitute the coordinates into the slope formula, and simplify.

The slope of $\overleftrightarrow{LM}$ is *undefined*.

$\overleftrightarrow{LM}$ is a vertical line.

Because $\overleftrightarrow{JK}$ and $\overleftrightarrow{LM}$ are two different vertical lines, $\overleftrightarrow{JK} \parallel \overleftrightarrow{LM}$.

▶ Since the slopes of parallel lines are the *same*, you can use this fact to write the equation of a line parallel to a given line.

- Find the slope of the given line.
- Substitute that information into *either* the slope-intercept form or the point-slope form of a linear equation to write the equation of a parallel line.

Write an equation of a line parallel to the graph of $2x - y = 3$ with $y$-intercept of $-5$.

$2x - y = 3$ ← Solve for $y$ to find the slope of the given line.

$2x - 2x - y = -2x + 3$ ← Use the Subtraction Property of Equality.

$\frac{-y}{-1} = \frac{-2x+3}{-1}$ ← Use the Division Property of Equality.

$y = 2x - 3$

The slope is 2 and the $y$-intercept of the parallel line is $-5$.

**Think**
Since the $y$-intercept is given, substitute the given information into the slope-intercept form.

$y = mx + b$ ← Use the slope-intercept form.

$y = 2x - 5$ ← Substitute the given values.

So the graph of $y = 2x - 5$ is parallel to the graph of $2x - y = 3$.

## Example

**1** Write an equation of a line parallel to the graph of $y = -\frac{1}{3}x + 2$ and passing through the point $(3, 2)$.

$y = -\frac{1}{3}x + 2$

The slope is $-\frac{1}{3}$.

**Think**
Since a point is given, substitute the given information into the point-slope form.

$y - y_1 = m(x - x_1)$ ← Use the point-slope form.

$y - 2 = -\frac{1}{3}(x - 3)$ ← Substitute the given values.

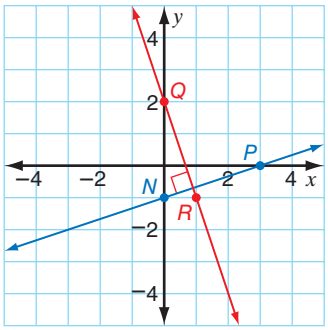
$y - 2 + 2 = -\frac{1}{3}x + 1 + 2$ ← Simplify; use the Addition Property of Equality.

$y = -\frac{1}{3}x + 3$

So the graph of $y = -\frac{1}{3}x + 3$ is parallel to the graph of $y = -\frac{1}{3}x + 2$.

Continue Lesson ➡

The lines shown in the graph at the right are perpendicular *because* they intersect to form right angles. The slopes of perpendicular lines are negative reciprocals of each other and their product is $-1$.




The equation of $\overleftrightarrow{NP}$ is $y = \frac{1}{3}x - 1$. Its slope is $\frac{1}{3}$.

The equation of $\overleftrightarrow{QR}$ is $y = -3x + 2$. Its slope is $-3$.

The product of the slopes is $\frac{1}{3}(-3) = -1$.

So $\overleftrightarrow{NP}$ is perpendicular to $\overleftrightarrow{QR}$.

**Key Concept**

**Perpendicular Lines**

- If the product of the slopes of two lines is $-1$, then the lines are perpendicular.
- If two lines are perpendicular and neither one is vertical, the product of their slopes is $-1$.
- Vertical lines are perpendicular to horizontal lines.

To determine whether lines are perpendicular:

- Find the slope of each line that is given.
- Compare the slopes of the lines. If the slopes are negative reciprocals, the lines are perpendicular.

Are the graphs of $y = \frac{4}{3}x - 1$ and $3x + 4y = -8$ perpendicular?

Write $3x + 4y = -8$ in slope-intercept form. Compare it with $y = \frac{4}{3}x - 1$.

$3x - 3x + 4y = -3x - 8$ ← Use the Subtraction Property of Equality.

$\frac{4y}{4} = \frac{-3x - 8}{4}$ ← Use the Division Property of Equality.

$y = -\frac{3}{4}x - 2$

The slope of $y = -\frac{3}{4}x - 2$ is $-\frac{3}{4}$. The slope of $y = \frac{4}{3}x - 1$ is $\frac{4}{3}$.

$\left(-\frac{3}{4}\right)\left(\frac{4}{3}\right) = -1$, so the slopes are negative reciprocals, and the lines are perpendicular.

## Examples

**1** Determine if the given lines are perpendicular.

$\overleftrightarrow{ST}$ is $y = 5x - 1$; $\overleftrightarrow{UV}$ is $10x + 2y = 12$.
Is $\overleftrightarrow{ST} \perp \overleftrightarrow{UV}$?

**Remember:**
$\perp$ is the symbol for "is perpendicular to."

Write $\overleftrightarrow{UV}$ in slope-intercept form. Compare it to $\overleftrightarrow{ST}$.

$10x + 2y = 12$

$10x - 10x + 2y = -10x + 12$ ← Use the Subtraction Property of Equality.

$\frac{-2y}{2} = \frac{-10x + 12}{2}$ ← Use the Division Property of Equality.

$y = -5x + 6$

The slope of $\overleftrightarrow{UV}$ is $-5$. The slope of $\overleftrightarrow{ST}$ is 5. $(-5)(5) \neq -1$
The slopes are not negative reciprocals, so the lines are not perpendicular.

**2** $l_1 \perp l_2$. If the slope of $l_1$ is 4 and the slope of $l_2$ is $\frac{x-6}{16}$, find the value of $x$.

Since $l_1 \perp l_2$, the slope of $l_2$ is $-\frac{1}{4}$, the negative reciprocal of 4, the slope of $l_1$.

**Solve:** $\frac{x-6}{16} = -\frac{1}{4}$ ← Write an equation to solve for $x$.

$4(x-6) = -16$ ← Apply the Cross-Products Rule.

$4x - 24 = -16$ ← Apply the Distributive Property.

$4x - 24 + 24 = -16 + 24$ ← Use the Addition Property of Equality.

$4x \div 4 = 8 \div 4$ ← Use the Division Property of Equality.

$x = 2$

**Check:** $\frac{x-6}{16} = -\frac{1}{4}$

$\frac{2-6}{16} \stackrel{?}{=} -\frac{1}{4}$

$\frac{-4}{16} \stackrel{?}{=} -\frac{1}{4}$

$-\frac{1}{4} = -\frac{1}{4}$ True

▶ There are many lines perpendicular to a given line. For each of them you can write an equation using the negative reciprocal of the slope of the given line.

- Use the slope-intercept form of the given line to find its slope. Find its negative reciprocal for the slope of the perpendicular line.
- Use this slope and the other information about the perpendicular line to write the equation.

Write an equation, in slope-intercept form, of the line that is perpendicular to the graph of $y = x + 2$ and that passes through the point $(-1, -4)$.

$y = 1x + 2$ ← Identify the slope: 1

The slope of the perpendicular line is $-1$. ← Find the negative reciprocal of 1.

$y - y_1 = m(x - x_1)$ ← Use the point-slope form.

$y - (-4) = -1[x - (-1)]$ ← Substitute the given values.

$y + 4 = -x - 1$ ← Simplify; then apply the Distributive Property.

$y + 4 - 4 = -x - 1 - 4$ ← Use the Addition Property of Equality.

$y = -x - 5$ ← Simplify.

So the equation of the perpendicular line is $y = -x - 5$.

## Try These

**Tell whether the pair of lines is *parallel*, *perpendicular*, or *neither*.**

**1.** $y = -\frac{5}{2}x + 7$ and $2x - 5y = 6$

**2.** Line 1 passes through $(0, 6)$ and $(-5, -1)$; line 2 passes through $(-1, 6)$ and $(4, -1)$.

**Find an equation of a line in slope-intercept form.**

**3.** parallel to $3x + 3y = -2$ with a $y$-intercept of 5

**4.** perpendicular to $y = -2x + 2$ and containing $(2, 1)$

**5. Discuss and Write** Can two parallel lines have the same $y$-intercept? Can two perpendicular lines have the same $y$-intercept? Explain.

Go to PRACTICE BOOK Lesson 5-6 for exercise sets.