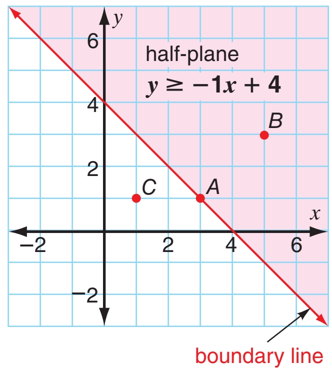
5-7 Graph a Linear Inequality in the Coordinate Plane

Objective To graph a linear inequality in two variables

The number sentence $y \geq -x + 4$ is a linear inequality in two variables. Its graph is shown at the right. The related equation, $y = -x + 4$, separates the coordinate plane into three sets of points: the points *on* the line, the points *above* the line, and the points *below* the line. The regions above and below the line are called half-planes. The line is the boundary of each half-plane.



A solution to a linear inequality in two variables is any ordered pair that makes the inequality true. To identify specific solutions, substitute any ordered pair located in the identified region or on the boundary line into the original linear inequality.

Point A (3, 1)	Point B (5, 3)	Point C (1, 1)
$1 \geq = -(3) + 4$	$3 \geq = -(5) + 4$	$1 \geq = -(1) + 4$
$1 \geq 1$ True	$3 \geq -1$ True	$1 \geq 3$ False

According to this graph, point A, (3, 1), and point B, (5, 3), are solutions of $y \geq -1x + 4$. Notice that point C, (1, 1), does *not* lie either in the shaded area or on the boundary line, and, therefore, it cannot be a solution of the inequality.

► To graph the solutions of a linear inequality:

- Write the inequality in slope-intercept form.
- Graph the boundary line by replacing the inequality sign with an equal sign. If the inequality is $\leq$ or $\geq$, the boundary is a *solid line*, which shows that the points on the line are included in the solution region. However, if the inequality is $<$ or $>$, the boundary is a *dashed line*, which shows that the points on the line are *not* part of the solution region.
- Shade the appropriate half-plane.
- Choose test points on the line, below the line, and above the line to verify that the shaded area represents the solution.

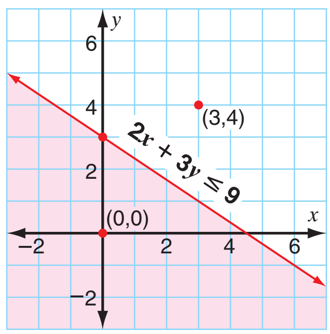
Solve: $2x + 3y \leq 9$ ← Solve for y and write in slope-intercept form.

$$3y \leq -2x + 9$$

$$y \leq -\frac{2}{3}x + 3$$

Graph: $y = -\frac{2}{3}x + 3$ ← Use a solid boundary line for $\leq$.

Shade the half-plane *below* the line.



Check: Check by choosing test points.

$3 \overset{?}{\leq} -\frac{2}{3}(0) + 3$	$0 \overset{?}{\leq} -\frac{2}{3}(0) + 3$	$4 \overset{?}{\leq} -\frac{2}{3}(3) + 3$
$3 \leq 3$ True	$0 \leq 3$ True	$4 \leq 1$ False

So the y-values in the solution include those that are both *equal to* and *less than* $-\frac{2}{3}x + 3$. Shade the half-plane that is *below* the line.

Examples

Graph each inequality.

1 $4x - 2y < -1$

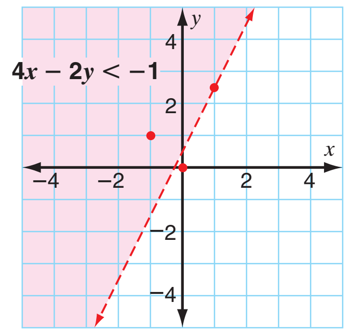
Solve: $-2y < -4x - 1$ ← Use the Subtraction Property of Inequality.

$y > 2x + \frac{1}{2}$ ← Use the Division Property of Inequality.

Remember: Reverse the inequality symbol when dividing by a negative number.

Graph: $y = 2x + \frac{1}{2}$ ← Use a dashed line for >.

Shade the half-plane *above* the line.



Check: Choose test points.

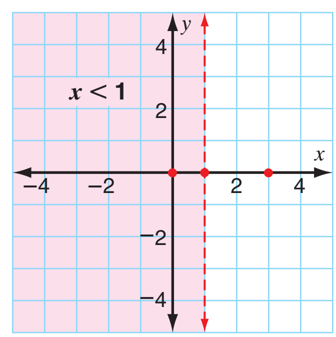
Point: $(0, 0)$	Point: $\left(1, 2\frac{1}{2}\right)$	Point: $(-1, 1)$
$0 \overset{?}{>} 2(0) + \frac{1}{2}$	$2\frac{1}{2} \overset{?}{>} 2(1) + \frac{1}{2}$	$1 \overset{?}{>} 2(-1) + \frac{1}{2}$
$0 > \frac{1}{2}$ False	$2\frac{1}{2} > 2\frac{1}{2}$ False	$1 > -2\frac{1}{2}$ True

Since the statement is false, the half-plane containing $(0, 0)$ should *not* be shaded.

2 $x < 1$

Graph: $x = 1$ ← Use a dashed line for <.

Shade the half-plane to the *left* of the line.



Check: Choose test points.

Point: $(0, 0)$	Point: $(1, 0)$	Point: $(3, 0)$
$0 < 1$ True	$1 < 1$ False	$3 < 1$ False

Since the statement is true, the half-plane containing $(0, 0)$ should be shaded.

Try These

Determine whether or not the ordered pair is a solution of the linear inequality.

1. $y > x + 3; (1, 4)$ **2.** $3y \geq -9; (-5, 2)$ **3.** $2x - y \leq 6; (4, -2)$ **4.** $3x + 2y < 9; \left(\frac{1}{2}, \frac{7}{2}\right)$

Write the equation of the boundary line in slope-intercept form.
Describe each line as *dashed* or *solid*.

5. $2y - x \leq 6$ **6.** $6x + 3y > 9$ **7.** $3x - y < 4$ **8.** $-2x + y \geq 0$

Graph each linear inequality.

9. $x + y > 5$ **10.** $2y \leq 5$ **11.** $3x \geq y$ **12.** $-3x - 6 > 0$

13. Discuss and Write Lee has \$220 to spend on shirts at \$22 each and pants at \$38 each. What inequality represents the possible number of shirts and pants that he can buy? Find at least three possible solutions. Can the solutions include negative numbers?

Go to PRACTICE BOOK Lesson 5-7 for exercise sets.