Name _____

5-8A Piecewise-Defined Functions

Objective To graph and find values for piecewise-defined functions that are defined over two adjacent domains

Cory goes hiking and fishing. He hikes for 2 hours at a rate of 3.5 miles per hour to go from his tent to the river. Then he stops and fishes for 2 hours. What is the distance, D, between Cory hikes and his tent for times t equal to 1.5 hours and 3.5 hours?

▶ Since Cory's trip is in two parts, hiking and fishing, both the function and graph also have two pieces. This type of function is called a **piecewise function.** For the first two hours of the trip, his distance can be found by multiplying the rate, 3.5 mi/h, by the time, t. Once he stops and fishes at the river, his distance from the tent stays the same at 7 miles.

You can represent a piecewise function in two different ways.

Key Concept
A **piecewise function** is a function that is defined by different expressions on different intervals.

Method 1: Algebraically

- Write the function as two pieces using a bracket. Each piece shows how to write the the function for a particular range of t values.

$$D(t) = \begin{cases} 3.5t & 0 \le t \le 2 \\ 7 & 2 < t \le 4 \end{cases}$$

- Evaluate the function for each value of t.

$D(1.5) = 3.5(1.5) = 5.25$ ← Use $D(t) = 3.5t$.

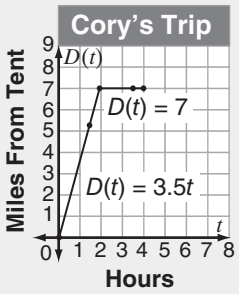
$D(3.5) = 7$ ← Use $D(t) = 7$.

Method 2: Graphically

- Draw the graph using each description. From $0 \le t \le 2$, draw a line with slope 3.5. From $2 < t \le 4$, draw a line with no slope.
- Use the graph to find the $d(t)$values for $t = 1.5$ hours and $t = 3.5$ hours

$D(1.5) = 5.25$

$D(3.5) = 7$



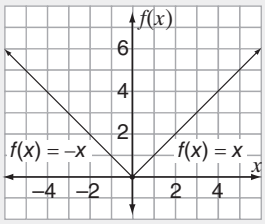
So, Cory is 5.25 miles from the tent after 1.5 hours and 7 miles from the tent after 3.5 hours.

▶ Absolute-value and step functions are also piecewise functions. Sometimes piecewise functions have segments that do not connect. Use an open circle for an endpoint not included in the domain, and a closed circle for an endpoint that is in the domain.

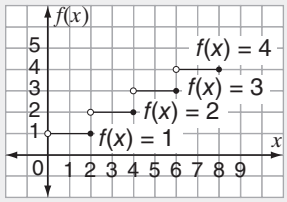
1 An absolute-value function is a piecewise function.

If $f(x) = |x|$, then

$$f(x) = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$$



2 A **step function** is a function that has a series of horizontal segments that increase or decrease regularly.



$$f(x) = \begin{cases} 1 & \text{if } 0 < x \le 2 \\ 2 & \text{if } 2 < x \le 4 \\ 3 & \text{if } 4 < x \le 6 \\ 4 & \text{if } 6 < x \le 8 \end{cases}$$

1. Discuss and Write Why is an absolute-value function a type of piecewise function?

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5-8A Piecewise-Defined Functions

Draw the graph of each function.

2. $f(x) = \begin{cases} 2x - 3 & \text{if } x \geq 2 \\ -2x + 3 & \text{if } x < 0 \end{cases}$

3. $g(x) = \begin{cases} 4 & \text{if } x \geq 1 \\ -x & \text{if } x < 1 \end{cases}$

4. $f(x) = \begin{cases} 2x - 2 & \text{if } x \geq 2 \\ -0.5x + 2 & \text{if } x < 2 \end{cases}$

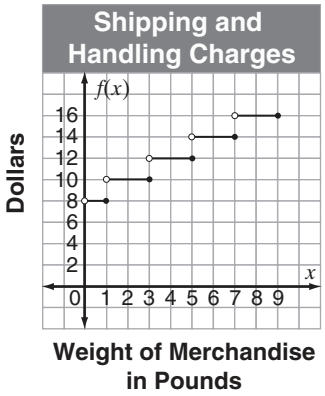
5. $g(x) = \begin{cases} x - 4 & \text{if } x \geq 2 \\ -x & \text{if } x < 2 \end{cases}$

6. $f(x) = \begin{cases} -1.5x + 4 & \text{if } x > 1 \\ 2 & \text{if } -1 \leq x \leq 1 \\ 1.5x + 1 & \text{if } x < -1 \end{cases}$

7. $g(x) = \begin{cases} 4 & \text{if } x > 2 \\ -x - 1 & \text{if } -4 \leq x \leq 2 \\ -2 & \text{if } x < -4 \end{cases}$

Problem Solving

Use the graph titled Shipping and Handling Charges for problems 8–10.



8. What is the shipping and handling charge for merchandise weighing 5.9 pounds?

_____

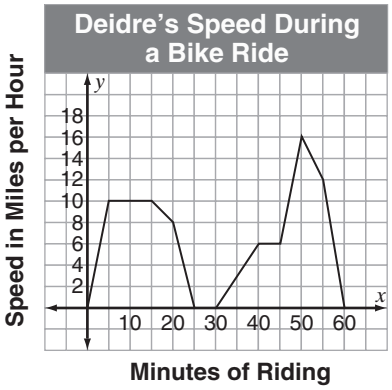
9. What pattern or rule is used to determine the shipping and handling charges?

_____

10. If the pattern continues, what would the shipping and handling charges be for a 13-pound box?

_____

Use the graph titled Deidre's Speed During a Bike Ride for problems 11–14.



11. What is the value of $f(20)$? What does $f(20)$ mean in terms of Deidre's bike ride?

_____

12. What is one interval in which the slope of the line is positive? What does a negative slope mean in terms of the bike ride?

_____

13. What is one interval in which the slope of the line is negative? What does a positive slope mean in terms of the bike ride?

_____

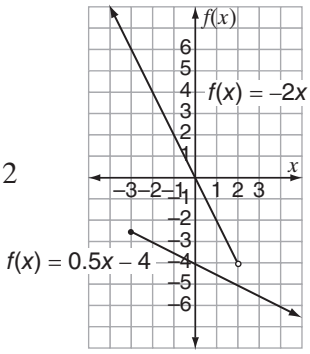
14. For which values of x is the value of $f(x) = 0$? What does $f(x) = 0$ mean in terms of the bike ride?

_____

EXPLAIN YOUR REASONING

15. Damien gave the equation and graph at the right as an example of a piecewise function. Is it a piecewise function? Explain.

$$f(x) = \begin{cases} 0.5x - 4 & \text{if } x \geq -2 \\ -2x & \text{if } x < 2 \end{cases}$$



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