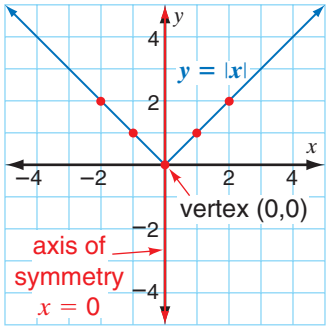
# 5-8 Absolute-Value Functions

**Objective** To graph the basic absolute-value function and identify its characteristics
- To investigate the graphs of absolute-value functions using a handheld device

A function rule that contains an absolute-value expression is called an absolute-value function. The graphs of absolute-value functions are not linear, although they are related to linear functions.




▶ To graph the absolute-value function $y = |x|$, make a function table, and graph the points.

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y = \|x\|$ | 2 | 1 | 0 | 1 | 2 |

The graph is V-shaped and is separated into two congruent parts by a line, called the axis of symmetry. The left and right parts are the graphs of $y = -x$ for $y \leq 0$ and $y = x$ for $y \geq 0$, respectively. The point in which the two parts meet is called the vertex.

An absolute-value graph shows:
- the vertex
- the axis of symmetry
- the $x$- and $y$-intercepts
- the domain
- the range

An absolute-value graph is a *function* because for every $x$-value, there is only one $y$-value.

The absolute-value graph above shows:
- The vertex is $(0, 0)$.
- The axis of symmetry is the $y$-axis ($x = 0$).
- The $x$- and $y$-intercepts are both 0.
- The domain ($x$-values) is the set of all real numbers.
- The range ($y$-values) is $y \geq 0$.

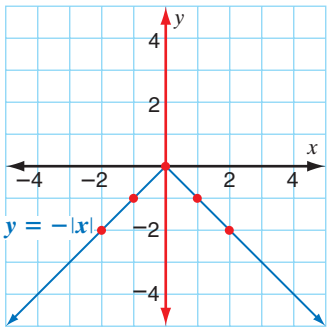
The graph of $y = |x|$ is a function because for every $x$-value, there is only one $y$-value.

▶ There are other absolute-value functions whose graphs are V-shaped. The graphs of these functions can *open up* or *down*.

Graph: $y = -|x|$
Identify the vertex, axis of symmetry, $x$- and $y$-intercepts, domain, and range. Tell whether the graph opens up or down.

Make a function table. Choose positive, negative, and zero values for $x$. Then graph.

| $x$ | $y = -\|x\|$ |
|---|---|
| −2 | −2 |
| −1 | −1 |
| 0 | 0 |
| 1 | −1 |
| 2 | −2 |



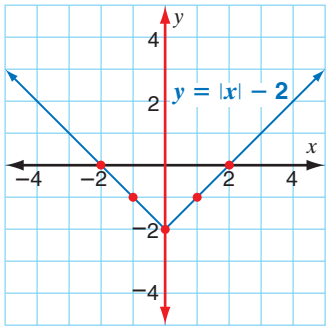

The absolute-value graph at the left shows the following:
- The vertex is $(0, 0)$.
- The axis of symmetry is $x = 0$.
- The $x$-intercept is 0.
- The $y$-intercept is 0.
- The domain is all real numbers.
- The range is $y \leq 0$.
- The graph opens down.

Notice when $y = |x|$, the graph of the absolute-value function opens up, and when $y = -|x|$, the graph of the absolute-value function opens down.

## Example

**1** Graph: $y = |x| - 2$
Choose positive, negative, and zero values for $x$. Then graph.

| $x$ | $y = \|x\| - 2$ |
|---|---|
| −2 | 0 |
| −1 | −1 |
| 0 | −2 |
| 1 | −1 |
| 2 | 0 |



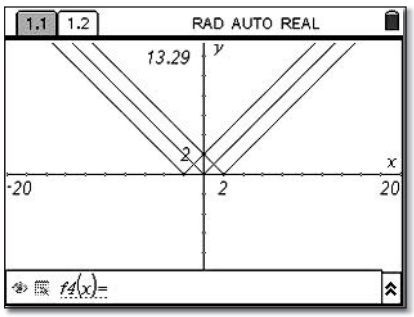

The absolute-value graph at the left shows the following:

- The vertex is $(0, -2)$.
- The axis of symmetry is $x = 0$.
- The $x$-intercepts are $-2, 2$.
- The $y$-intercept is $-2$.
- The domain is all real numbers.
- The range is $y \geq -2$.
- The graph opens up.

▶ You can graph various absolute-value functions such as $f(x) = |x|, f(x) = |x + c|$, and $f(x) = |x - c|$, on the same coordinate plane using a handheld. This device can make exploring each graph's unique and similar characteristics easier.

Graph $y = |x|, y = |x + 2|$, and $y = |x - 2|$ on the same coordinate plane using a handheld device.

To enter the equations, press [home], select [2]: Graphs and Geometry, and then input each function separately into $f_1(x)$, $f_2(x)$, and $f_3(x)$.




| | $y = \|x\|$ | $y = \|x + 2\|$ | $y = \|x - 2\|$ |
|---|---|---|---|
| **vertex** | (0, 0) | (−2, 0) | (2, 0) |
| **axis of symmetry** | $x = 0$ | $x = -2$ | $x = 2$ |
| ***x*-intercept(s)** | 0 | −2 | 2 |
| ***y*-intercept** | 0 | 2 | 2 |
| **domain** | all real numbers | | |
| **range** | $y \geq 0$ | $y \geq 0$ | $y \geq 0$ |

Notice that when $c$ is 0, the axis of symmetry is $x = 0$; when $c$ is $+2$, the axis of symmetry is $x = -2$; and when $c$ is $-2$, the axis of symmetry is $x = 2$.

## Try These

**Identify the vertex by graphing the absolute-value function.**

**1.** $y = |x - 1|$ **2.** $y = |x| - 1$ **3.** $y = |x + 1|$ **4.** $y = |x| + 1$

**Graph the absolute-value function. Identify the vertex, axis of symmetry, *x*- and *y*-intercepts, domain, and range. Tell whether the graph opens up or down.**

**5.** $y = -|x + 3|$ **6.** $y = |x - 2| + 1$ **7.** $y = -|x| - 2$

**8. Discuss and Write** Use a handheld to graph $y = |x|, y = |\frac{1}{2}x|$, and $y = |5x|$. What is the same and what is different about the graphs? Describe how changing the coefficient of $x$ affects the graph.

Go to PRACTICE BOOK Lesson 5-8 for exercise sets.