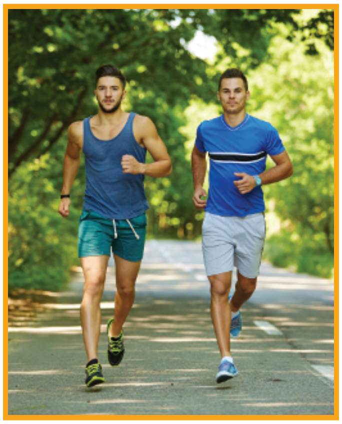
6-1 Solve Systems of Linear Equations Graphically

Objective To solve systems of linear equations in two variables graphically

Problem-Solving Strategy 8

Max challenges his brother Eru to a race on the way to football practice. He gives Eru a 20-yard head start. If Max runs 6 yards per second and Eru runs 4 yards per second, how long will it take Max to catch up to Eru?



To find how long it would take Max to catch up to Eru, write and solve two equations that represent the situation.

► A set of two or more equations that have variables in common is a system of linear equations or simultaneous equations.

Let y = the distance (in yards).
Let x = time (in seconds).

Think distance = rate • time

$$\begin{cases} y = 6x & \leftarrow \text{Max's position at time } x \\ y = 4x + 20 & \leftarrow \text{Eru's position at time } x \end{cases}$$

► A solution of a system of equations is a set of values for the variables that make each equation in the system true.

To find a solution, you can use a graph. A graph can tell you what *appears* to be the solution, or the point that both lines have in common. This point is called the *point of intersection.*

Graph each of the equations on the same coordinate plane by making a function table, or by using the slope and y-intercept for each equation. Locate the point where the lines intersect to find the solution of the system.

Method 1 Make a function table for each equation.

$y = 6x$

x	y
0	0
5	30
8	48

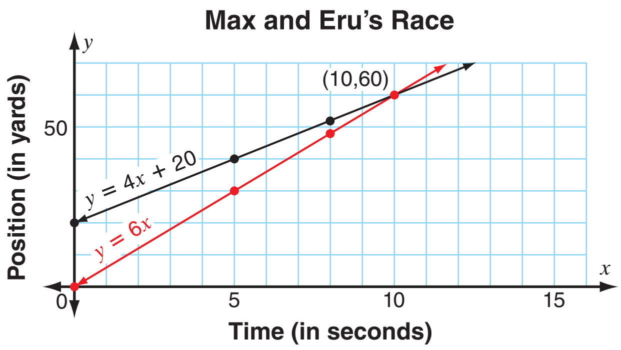
$y = 4x + 20$

x	y
0	20
5	40
8	52

Method 2 Use the slope and y-intercept.

$y = 6x \leftarrow$ slope $= \frac{6}{1}$, y-intercept $= 0$

$y = 4x + 20 \leftarrow$ slope $= \frac{4}{1}$, y-intercept $= 20$



Check: Substitute the values for the variables into each equation, and verify that the solution, (10, 60), satisfies *both* equations.

$y = 6x$	$y = 4x + 20$
$60 \stackrel{?}{=} 6(10)$	$60 \stackrel{?}{=} 4(10) + 20$
$60 = 60$ True	$60 = 60$ True

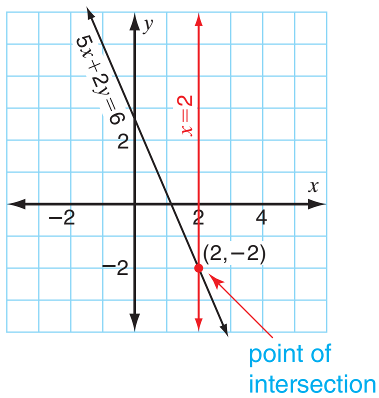
The lines appear to intersect at (10, 60).

So it will take Max 10 seconds to catch up to Eru.

Example

1 Solve the system by graphing: $\begin{cases} 5x + 2y = 6 \\ x = 2 \end{cases}$

Use the slope and y-intercept to graph each equation. Then locate the point of intersection.



Graph: $5x + 2y = 6$ ← Solve for y to write the equation in slope-intercept form.

$$5x - 5x + 2y = 6 - 5x$$

$$\frac{2y}{2} = \frac{6 - 5x}{2}$$

$$y = \frac{6}{2} - \frac{5x}{2}$$

$$y = 3 - \frac{5}{2}x$$

$$y = -\frac{5}{2}x + 3$$ ← The slope is $-\frac{5}{2}$, and the y-intercept is 3.

Graph: $x = 2$ ← This is a vertical line.

$(2, -2)$ ← This is the point where the two lines appear to intersect.

Check: Substitute the solution into each of the original equations.

$5x + 2y = 6$ | $x = 2$

$5(2) + 2(-2) \stackrel{?}{=} 6$ | $2 = 2$ True

$10 - 4 \stackrel{?}{=} 6$

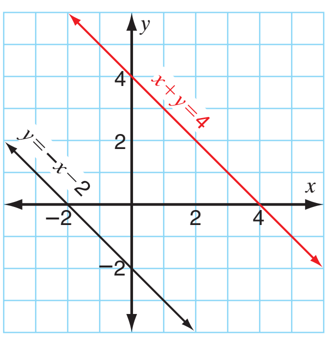
$6 = 6$ True

So the solution of the system of equations is $(2, -2)$.

▶ A system of equations can have no solution. This occurs when two lines do *not* intersect, or are parallel.

Solve the system by graphing: $\begin{cases} x + y = 4 \\ y = -x - 2 \end{cases}$

Use the slope and y-intercept to graph each equation. Then locate the point of intersection.



Graph: $x + y = 4$ ← Solve for y.

$$y = -x + 4$$ ← The slope is -1, and the y-intercept is 4.

Graph: $y = -x - 2$ ← The slope is -1, and the y-intercept is -2.

The lines have the same slope and different y-intercepts. They are parallel lines and do not intersect. There is no ordered pair that makes both of the equations true. The system has *no solution*.

Continue Lesson ➡

► A system of equations can also have infinitely many solutions.

Solve the system by graphing: $\begin{cases} x + 2y = 2 \\ -3x - 6y = -6 \end{cases}$

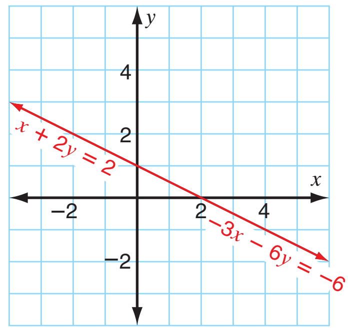
Use the slope and y-intercept to graph each equation. Then locate the point of intersection.

Graph: $x + 2y = 2$ ← Solve for y to write the equation in slope-intercept form.

$y = -\frac{1}{2}x + 1$ ← The slope is $-\frac{1}{2}$, and the y-intercept is 1.

Graph: $-3x - 6y = -6$ ← Solve for y.

$y = -\frac{1}{2}x + 1$ ← The slope is $-\frac{1}{2}$, and the y-intercept is 1.



The graphs coincide. The equations are equivalent; they describe the same line. Every point *on the line* is a solution of both equations. There are infinitely many solutions for this system of equations.

Example

1 Solve the system by graphing: $\begin{cases} 6x - 10y = -20 \\ -\frac{3}{5}x = 2 - y \end{cases}$

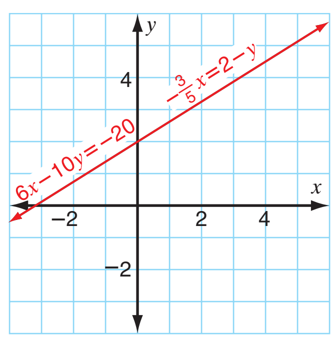
Use the slope and y-intercept to graph each equation. Then locate the point of intersection.

Graph: $6x - 10y = -20$ ← Solve for y.

$y = \frac{3}{5}x + 2$ ← The slope is $\frac{3}{5}$, and the y-intercept is 2.

Graph: $-\frac{3}{5}x = 2 - y$ ← Solve for y.

$y = \frac{3}{5}x + 2$ ← The slope is $\frac{3}{5}$, and the y-intercept is 2.

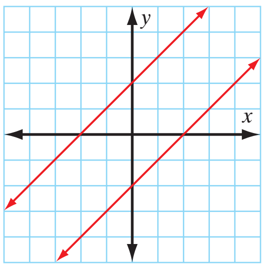
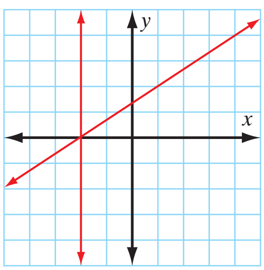
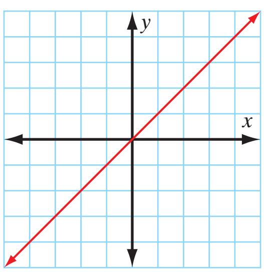


The graphs *coincide*. There are infinitely many solutions for the system.

► The terms *consistent*, *inconsistent*, *independent*, and *dependent* are used to describe systems of equations.

- If there is at least one solution for a system of equations, the system is consistent.
- If there are no solutions, the system is inconsistent.
- If a system of equations describes two different lines that intersect, the system is independent.
- If both equations describe the same line, the system is dependent.

The chart below summarizes the three types of systems of equations.

Systems of Equations			
Graphs of the Equations		**Number of Solutions**	**Description of System**
	parallel lines	none	inconsistent
	intersecting lines	exactly one	consistent and independent
	same line	infinitely many	consistent and dependent

Try These

Graph each system of equations. Find the number of solutions, and then describe the system.

1. $\begin{cases} 6x - 4y = -36 \\ 9x - 54 = 6y \end{cases}$

2. $\begin{cases} 2x + 3y = 2 \\ y = -x + 2 \end{cases}$

3. $\begin{cases} 2y - 10 = -\frac{5}{2}x \\ 15x - 60 = -12y \end{cases}$

4. $\begin{cases} 2x - y = -8 \\ y = 4 \end{cases}$

5. Nasim wants to compare the membership costs of two gyms. Gym I requires a \$120 membership fee and then charges \$20 each month. Gym II has no membership fee but charges \$30 a month. In what month will the cost be the same?

6. **Discuss and Write** How can you determine whether two lines will intersect without graphing the lines? Explain.

Go to PRACTICE BOOK Lesson 6-1 for exercise sets.