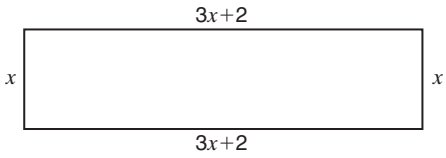
6-2 Solve Systems of Linear Equations by Substitution

Objective To solve systems of linear equations in two variables algebraically by substitution

The length of a rectangle is 2 cm more than 3 times its width. If the perimeter of the rectangle is 28 cm, find its dimensions.



To find the dimensions of the rectangle, write and solve a system of linear equations.

Let x = width of the rectangle.
Let y = length of the rectangle.

$$\begin{cases} y = 3x + 2 \\ 2x + 2y = 28 \end{cases}$$

$y = 3x + 2$ ← The length (y) is 2 cm *more than* 3 *times* its width.

$2x + 2y = 28$ ← The perimeter is 28 cm.

One way to find an exact solution to a system of linear equations is to use substitution. To use this method, solve one of the equations for one of the variables and then substitute the expression for the variable into the second expression.

Solve: $\begin{cases} y = 3x + 2 \\ 2x + 2y = 28 \end{cases}$

Substitute: $3x + 2$ for y into the second equation.

$$\begin{aligned} 2x + 2y &= 28 \\ 2x + 2(3x + 2) &= 28 && \leftarrow \text{Substitute } 3x + 2 \text{ for } y. \\ 2x + 6x + 4 &= 28 && \leftarrow \text{Apply the Distributive Property.} \\ 8x + 4 &= 28 && \leftarrow \text{Combine like terms.} \\ 8x + 4 - 4 &= 28 - 4 && \leftarrow \text{Use the Subtraction Property of Equality.} \\ 8x &= 24 \\ 8x \div 8 &= 24 \div 8 && \leftarrow \text{Use the Division Property of Equality.} \\ x &= 3 \end{aligned}$$

To find the corresponding y-value, substitute 3 for x into one of the equations, then solve.

$$\begin{aligned} y &= 3x + 2 \\ y &= 3(3) + 2 && \leftarrow \text{Substitute 3 for } x. \\ y &= 11 \end{aligned}$$

Check: Substitute 3 for x and 11 for y into the original equations.

$$\begin{aligned} y &= 3x + 2 \\ 11 &\stackrel{?}{=} 3(3) + 2 \\ 11 &= 11 \text{ True} \end{aligned}$$

$$\begin{aligned} 2x + 2y &= 28 \\ 2(3) + 2(11) &\stackrel{?}{=} 28 \\ 6 + 22 &\stackrel{?}{=} 28 \\ 28 &= 28 \text{ True} \end{aligned}$$

So the length of the rectangle is 11 cm, and the width is 3 cm.

Sometimes you need to transform one of the equations to solve for one of the variables. The equation that is easier to solve is the one with a variable that has a coefficient of 1.

Solve: $\begin{cases} x + 4y = 7 \\ 2x - 2y = 9 \end{cases}$

$$x + 4y - 4y = 7 - 4y \quad \leftarrow \text{Use the Subtraction Property of Equality.}$$
$$x = 7 - 4y$$

Substitute: Replace $7 - 4y$ for x in the second equation, and solve for y. To find the corresponding x-value, substitute the value you determine for y into one of the equations.

$$2x - 2y = 9$$
$$2(7 - 4y) - 2y = 9 \quad \leftarrow \text{Substitute } 7 - 4y \text{ for } x.$$
$$14 - 8y - 2y = 9 \quad \leftarrow \text{Use the Distributive Property.}$$
$$14 - 10y = 9 \quad \leftarrow \text{Combine like terms.}$$
$$14 - 14 - 10y = 9 - 14 \quad \leftarrow \text{Use the Subtraction Property of Equality.}$$
$$\frac{-10y}{-10} = \frac{-5}{-10} \quad \leftarrow \text{Use the Division Property of Equality.}$$
$$y = \frac{1}{2}$$

$$x + 4\left(\tfrac{1}{2}\right) = 7 \quad \leftarrow \text{Substitute } \tfrac{1}{2} \text{ for } y.$$
$$x + 2 = 7$$
$$x + 2 - 2 = 7 - 2 \quad \leftarrow \text{Use the Subtraction Property of Equality.}$$
$$x = 5$$

Check: Substitute 5 for x and $\frac{1}{2}$ for y into *both* equations.

$$x + 4y = 7 \qquad 2x - 2y = 9$$
$$5 + 4\left(\tfrac{1}{2}\right) \stackrel{?}{=} 7 \qquad 2(5) - 2\left(\tfrac{1}{2}\right) \stackrel{?}{=} 9$$
$$5 + 2 \stackrel{?}{=} 7 \qquad 10 - 1 \stackrel{?}{=} 9$$
$$7 = 7 \text{ True} \qquad 9 = 9 \text{ True}$$

Solution: $\left(5, \frac{1}{2}\right)$ ← Write the solution as an ordered pair.

Try These

Solve each system of equations by using the substitution method.

1. $\begin{cases} x + 2y = 14 \\ 3x + 5y = 40 \end{cases}$

2. $\begin{cases} 3x - 6y = 0 \\ 2x - 2y = 1 \end{cases}$

3. $\begin{cases} 3x + 2y = -12 \\ 0.2x - 0.4y = -1.4 \end{cases}$

4. A box containing 3 dictionaries and 8 atlases weighs 35 pounds. Each dictionary weighs twice as much as an atlas. How much does each type of book weigh?

5. Discuss and Write If you were asked to solve $\begin{cases} 2x + 4y = -6 \\ 3x - y = 5 \end{cases}$ using substitution, which equation would you choose to solve for x or y? Explain.

Go to PRACTICE BOOK Lesson 6-2 for exercise sets.