6-3 Solve Systems of Linear Equations by Elimination

Objective To solve systems of linear equations in two variables algebraically by using addition or subtraction to eliminate one variable

Tanisha pays $49 for one pair of jeans and two shirts. The next day, she returns the two shirts for store credit, buys two more pairs of the same jeans, and pays an additional $26. What is the cost of one pair of jeans? What is the cost of one shirt?

To find the cost of one pair of jeans and one shirt, write and solve a system of linear equations.

Let j = the cost of a pair of jeans.

Let s = the cost of a shirt.

$$\begin{cases} j + 2s = 49 & \leftarrow \text{The cost of 1 pair of jeans and 2 shirts is \$49.} \\ 2j - 2s = 26 & \leftarrow \text{The cost of 2 pairs of jeans less a credit for 2 shirts is \$26.} \end{cases}$$

Another way to solve a system of equations is by eliminating one variable by using addition or subtraction. When the coefficients of the same variable are *opposites*, you can *add* the two equations to eliminate one variable.

Key Concept

Addition Property of Equality:
For all numbers a, b, c, and d,
if $a = b$ and $c = d$, then $a + c = b + d$.

1 Add the equations.

$$\begin{array}{rl} j + 2s = 49 & \\ \underline{2j - 2s = 26} & \leftarrow \begin{array}{l} +2s \text{ and } -2s \text{ are opposites.} \\ \text{Use the Addition Property of Equality.} \end{array} \\ 3j = 75 & \\ \frac{3j}{3} = \frac{75}{3} & \leftarrow \text{Use the Division Property of Equality.} \\ j = 25 & \end{array}$$

2 Substitute the value of the variable into one of the original equations. Solve for the other variable.

$$\begin{array}{rl} 25 + 2s = 49 & \leftarrow \text{Solve for } s \text{ by substituting 25 for } j. \\ 25 - 25 + 2s = 49 - 25 & \leftarrow \text{Use the Subtraction Property of Equality.} \\ 2s = 24 & \\ \frac{2s}{2} = \frac{24}{2} & \leftarrow \text{Use the Division Property of Equality.} \\ s = 12 & \end{array}$$

3 Check the answer by substituting the values of j and s into *both* equations.

$$\begin{array}{rl} j + 2s = 49 & \\ 25 + 2(12) \stackrel{?}{=} 49 & \\ 25 + 24 \stackrel{?}{=} 49 & \\ 49 = 49 & \text{True} \end{array} \qquad \begin{array}{rl} 2j - 2s = 26 & \\ 2(25) - 2(12) \stackrel{?}{=} 26 & \\ 50 - 24 \stackrel{?}{=} 26 & \\ 26 = 26 & \text{True} \end{array}$$

So one pair of jeans costs $25, and one shirt costs $12.

If the coefficients of one of the variables are *the same* rather than opposites, you can *subtract* the systems of equations to eliminate one of the variables.

Steven usually walks or bikes when he wants to go somewhere. Last week, he walked for 2.5 hours and biked for 5 hours, traveling a total of 55 miles. This week, he walked for 2.5 hours and biked for 2 hours, traveling a total of 28 miles. Assume his walking and riding speeds are the same each week. What is his rate of speed while walking? While biking?

Let w = the rate at which Steven walks.
Let b = the rate at which Steven bikes.

Remember:
rate • time = distance

$$\begin{cases} 2.5w + 5b = 55 & \leftarrow \text{last week's distance} \\ 2.5w + 2b = 28 & \leftarrow \text{this week's distance} \end{cases}$$

1 Subtract the equations.

$$\begin{array}{rl} 2.5w + 5b = & 55 \\ -2.5w - 2b = & -28 \quad \leftarrow \text{To subtract, add the opposite of each term.} \\ \hline 3b = & 27 \end{array}$$

$3b \div 3 = 27 \div 3$ ← Use the Division Property of Equality.

$b = 9$

2 Substitute and solve for the other variable.

$2.5w + 2b = 28$

$2.5w + 2(9) = 28$ ← Substitute 9 for b.

$2.5w + 18 = 28$

$2.5w + 18 - 18 = 28 - 18$ ← Use the Subtraction Property of Equality.

$2.5w \div 2.5 = 10 \div 2.5$ ← Use the Division Property of Equality.

$w = 4$

3 Check by substituting the values of both variables into *both* equations.

$2.5w + 5b = 55$
$2.5(4) + 5(9) \stackrel{?}{=} 55$
$10 + 45 \stackrel{?}{=} 55$
$55 = 55$ True

$2.5w + 2b = 28$
$2.5(4) + 2(9) \stackrel{?}{=} 28$
$10 + 18 \stackrel{?}{=} 28$
$28 = 28$ True

So Steven walks 4 mph and bikes 9 mph.

Try These

Solve each system of equations by addition or subtraction.

1. $\begin{cases} x - y = 5 \\ 3x - y = 7 \end{cases}$

2. $\begin{cases} 3a + 2b = 0 \\ -4a + 2b = 7 \end{cases}$

3. $\begin{cases} 3r + 2s = -\frac{1}{2} \\ -2r - 2s = \frac{1}{6} \end{cases}$

4. $\begin{cases} 2x - 0.8 = -y \\ 2y + 1.4 = 2x \end{cases}$

5. Discuss and Write Explain how to solve a system of equations using elimination. Give an example of how to use subtraction.

Go to PRACTICE BOOK Lesson 6-3 for exercise sets.