6-4 Solve Equivalent Systems of Linear Equations

Objective To solve systems of linear equations in two variables using equivalent systems

Sometimes a system of linear equations cannot immediately be solved using the elimination method by addition or subtraction. You may need to make an equivalent system, a system that has the same solution set, by multiplication.

Consider the system:

$$\begin{cases} 2x + y = 2 \\ 5x - 3y = 27 \end{cases}$$

Think
Adding or subtracting the equations will *not* eliminate a variable.

► To solve the system, multiply one or both equations by a constant so that the coefficients of one of the variables are opposites.

- First, multiply the first equation by 3.

$$3(2x + y = 2) \longrightarrow 6x + 3y = 6$$

Think
Multiply the first equation by 3 so that when added to the second equation, the y terms are eliminated.

So the systems $\begin{cases} 2x + y = 2 \\ 5x - 3y = 27 \end{cases}$ and $\begin{cases} 6x + 3y = 6 \\ 5x - 3y = 27 \end{cases}$ are equivalent and will have the *same solution set*.

- Then solve this system by elimination, since the coefficients of y are now opposites.

$$\begin{aligned} 6x + 3y &= 6 \\ 5x - 3y &= 27 \\ \hline 11x \quad &= 33 \\ \frac{11x}{11} &= \frac{33}{11} \\ x &= 3 \end{aligned}$$

← Eliminate the y variable. Use the Addition Property of Equality.

← Use the Division Property of Equality.

- Now, solve for y.

$$\begin{aligned} 2(3) + y &= 2 \\ 6 + y &= 2 \\ 6 - 6 + y &= 2 - 6 \\ y &= -4 \end{aligned}$$

← Solve for y by substituting 3 for x.

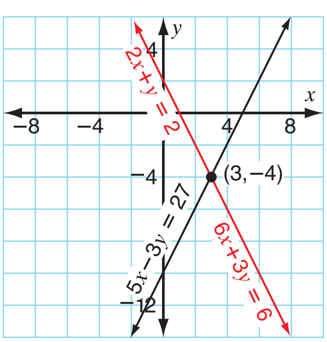
← Use the Subtraction Property of Equality.

Check: Substitute (3, 4) into the original equations.

$$\begin{aligned} 2x - y &= 2 \\ 2(3) - 4 &\stackrel{?}{=} 2 \\ 6 - 4 &\stackrel{?}{=} 2 \\ 2 &= 2 \text{ True} \end{aligned} \qquad \begin{aligned} 5x - 3y &= 27 \\ 5(3) - 3(-4) &\stackrel{?}{=} 27 \\ 15 + 12 &\stackrel{?}{=} 27 \\ 27 &= 27 \text{ True} \end{aligned}$$

So the solution for the system is (3, −4).

► You can use a graph to test that the solution is reasonable. Look at the graph at the right. The solution of the equivalent systems by elimination is the same as the solution by graphing.



▶ Sometimes both of the equations must be multiplied by a nonzero number to get an equivalent system with opposite coefficients for one variable.

PROBLEM-SOLVING STRATEGY 8

Solve $\begin{cases} 6x - 4y = 14 \\ 9x + 5y = -1 \end{cases}$

Think

The LCM of 6 and 9 is 18.
The LCM of 4 and 5 is 20.
The coefficients of y have opposite signs, so one way to find the solutions is to multiply the first equation by 5 and the second equation by 4.

$5(6x - 4y = 14)$
$4(9x + 5y = -1)$ ← Use the Multiplication Property of Equality to get an equivalent system with opposite coefficients for one of the variables.

$30x - 20y = 70$
$36x + 20y = -4$ ← Eliminate the y variable. Since the coefficients of y are opposites, the system can now be solved by elimination.

$$\begin{array}{r} 30x - 20y = 70 \\ 36x + 20y = -4 \\ \hline 66x \qquad = 66 \end{array}$$
← Use the Addition Property of Equality to combine equations.

$66x \div 66 = 66 \div 66$ ← Use the Division Property of Equality.

$x = 1$

$6(1) - 4y = 14$ ← Solve for y by substituting 1 for x.

$6 - 6 - 4y = 14 - 6$ ← Use the Subtraction Property of Equality.

$-4y \div (-4) = 8 \div (-4)$ ← Use the Division Property of Equality.

$y = -2$

Check: Substitute the solution in the original equations.

$6x - 4y = 14$	$9x + 5y = -1$
$6(1) - 4(-2) \stackrel{?}{=} 14$	$9(1) + 5(-2) \stackrel{?}{=} -1$
$6 + 8 \stackrel{?}{=} 14$	$9 - 10 \stackrel{?}{=} -1$
$14 = 14$ True	$-1 = -1$ True

So the solution for the system is $(1, -2)$.

Try These

Solve each system of equations by using substitution or elimination.

1. $\begin{cases} 3x + 6y = 3 \\ 4x = 8y - 5 \end{cases}$

2. $\begin{cases} 4a - 2b = 10 \\ 3a + 5b = -18\frac{1}{2} \end{cases}$

3. $\begin{cases} 5x = y + 8 \\ -x + 2y = -7 \end{cases}$

4. $\begin{cases} 2x - 3y = 0 \\ 3x - 2y = 1 \end{cases}$

5. Discuss and Write How would you solve the following system: $\begin{cases} 3x + 2y = 7 \\ 2y - 1 = 5x \end{cases}$?

Would you use the substitution or the elimination method? Show how you would set up the system, and explain why you chose that method.

Go to PRACTICE BOOK Lesson 6-4 for exercise sets.