6-5 Apply Systems of Linear Equations

Objective To solve a variety of verbal problems using systems of linear equations in two variables

Alan bought 5 pens, some blue pens for \$0.70 each and some red pens for \$.65 each. If he received \$1.65 change from \$5.00, how many red pens did he buy?

Let b = the number of blue pens.
Let r = the number of red pens.

$$\begin{cases} 0.70b + 0.65r = 3.35 & \leftarrow \text{The total cost was \$5.00 − \$1.65, or \$3.35.} \\ b + r = 5 & \leftarrow \text{Alan bought a total of 5 pens.} \end{cases}$$

$100(0.70b + 0.65r) = 100(3.35)$ ← Multiply by 100 so that the coefficients of the variables are whole numbers.

$$70b + 65r = 335$$

Since the second equation, $b + r = 5$, is easily solved for b or r, substitution is a convenient way to solve this system.

- Solve the second equation for b.

$$\begin{aligned} b + r &= 5 \\ b + r - r &= 5 - r \quad \leftarrow \text{Use the Subtraction Property of Equality.} \\ b &= 5 - r \end{aligned}$$

- Substitute the value for b into the other equation.

$$\begin{aligned} 70b + 65r &= 335 \\ 70(5 - r) + 65r &= 335 \quad \leftarrow \text{Apply the Distributive Property.} \\ 350 - 70r + 65r &= 335 \quad \leftarrow \text{Combine like terms.} \\ 350 - 5r &= 335 \quad \leftarrow \text{Use the Subtraction Property of Equality.} \\ -5r &= -15 \quad \leftarrow \text{Use the Division Property of Equality.} \\ r &= 3 \end{aligned}$$

So Alan bought 3 red pens.

Examples

1 PROBLEM-SOLVING STRATEGY 3

Two river towns are 60 miles apart. A boat can cover the 60 miles in 4 hours when traveling with the current. Traveling against the current, the boat takes 6 hours. What is the speed of the current? the speed of the boat?

Let b = the speed of the boat.
Let c = speed of the current.

	Rate (mph)	•	Time (hrs)	=	distance
With the current	$b + c$	•	4	=	60
Against the current	$b - c$	•	6	=	60

$$\begin{cases} 4b + 4c = 60 \\ 6b - 6c = 60 \end{cases}$$

- You can use elimination to solve this system by multiplying the first equation by 3 and the second equation by 2.

$$\begin{aligned} 3(4b + 4c = 60) &\rightarrow \quad 12b + 12c = 180 \\ 2(6b - 6c = 60) &\rightarrow \quad 12b - 12c = 120 \\ \hline 24b &\qquad\quad = 300 \\ b &= 12.5 \end{aligned}$$

The speed of the boat is 12.5 miles per hour.

- Substitute the value of b into either original equation.

$$\begin{aligned} 4(12.5) + 4c &= 60 \\ 50 + 4c &= 60 \\ 4c &= 10 \\ c &= 2.5 \end{aligned}$$

The speed of the current is 2.5 mph.

2 How much pure bleach and how much of a 20% bleach solution must be mixed to get 4 liters of a 30% bleach solution?

Think
Pure bleach means 100% bleach.

Let x = the number of liters of 20% bleach solution.
Let y = the number of liters of pure bleach.

	20% solution	+	pure bleach	=	30% solution
Amount of solution (L)	x	+	y	=	4
Amount of bleach (L)	$0.20x$	+	y	=	0.30(4)

$$\begin{cases} x + y = 4 \\ 0.20x + y = 1.20 \end{cases}$$

One way to solve this system is by subtraction.

$$\begin{array}{rcl} x + y = 4 & \longrightarrow & x + y = 4 \\ -(0.20x + y = 1.20) & \longrightarrow & -0.20x - y = -1.20 \\ & & \hline 0.80x = 2.80 \\ & & x = 3.5 \end{array}$$

Substitute the value of x into either one of the original equations.

$$\begin{array}{l} x + y = 4 \\ 3.5 + y = 4 \\ y = 0.5 \end{array}$$

So for a 30% bleach solution, mix 3.5 liters of a 20% bleach solution and 0.5 liter of pure bleach.

Try These

Solve each problem by writing and solving a system of linear equations.

1. Brianna has \$3.30 in dimes and quarters. The number of dimes she has is 3 fewer than twice the number of quarters. How many of each coin does she have?

2. In 5 years, Nick will be $\frac{3}{4}$ as old as his cousin. Three years ago, he was half as old as his cousin. How old are Nick and his cousin now?

3. Alma invested a total of \$4000. She put one in certificates of deposit that paid 5% annual interest and the rest in a money market account that paid 7% annual interest. The interest for 1 year totaled \$248. How much money did she invest in each type of account?

4. Todd charges twice as much for tutoring as he does for baby-sitting. One week, he earned \$105 for tutoring 4 hours and for baby-sitting 6 hours. How much does he charge for each activity?

5. Discuss and Write The following problem can be solved using one equation in one variable or a system of linear equations.

The length of a rectangle is twice the width. Its perimeter is 30 centimeters. Find the length and width.

Show how to set up and solve the problem using each method. Discuss the advantages of each one.

Go to PRACTICE BOOK Lesson 6-5 for exercise sets.