6-6

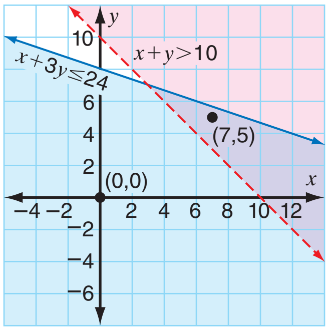
Graph Systems of Linear Inequalities

Objective To graph and solve systems of linear inequalities in two variables

A system of linear inequalities is a set of two or more linear inequalities with the same variables. A graph of the system shows all of its solutions. A solution makes each inequality in the system true.

Solve by graphing: $\begin{cases} x + y > 10 \\ x + 3y \leq 24 \end{cases}$

► To solve a system of inequalities by graphing:



- Graph the first inequality.

Remember: The graph of a linear inequality is a shaded half-plane.

$x + y > 10$

$y > -x + 10$ ← Solve the inequality for y. $m = -1$, $b = 10$

The solution lies *above* the boundary line $y = -x + 10$.

- Graph the second inequality.

$x + 3y \leq 24$

$y \leq -\frac{1}{3}x + 8$ ← Solve the inequality for y. $m = -\frac{1}{3}$, $b = 8$

The solution *includes* the boundary line $y = -\frac{1}{3}x + 8$ *and* all points *below* the boundary line.

The coordinates of the points in the pink region make the inequality $x + y > 10$ true. The coordinates of the points in the blue region *and* on the blue line makes the inequality $x + 3y \leq 24$ true. The coordinates of the points in the region where the graphs of the two inequalities *intersect* (the purple region and part of the blue line) make *both* inequalities true and are solutions to the system.

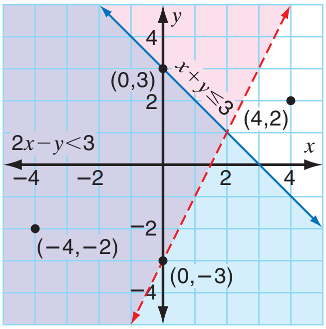
According to the graph, (7, 5) appears to be an ordered pair in the solution set for this system, and (0, 0) is not.

- Check the solution. Substitute the points above into *both* inequalities.

Ordered Pair	$x + y > 10$	$x + 3y \leq 24$	Solution
(7, 5)	$7 + 5 \overset{?}{>} 10$ $12 > 10$ True	$7 + 3(5) \overset{?}{\leq} 24$ $7 + 15 \overset{?}{\leq} 24$ $22 \leq 24$ True	(7, 5) is a solution to the system because it *satisfies* both inequalities.
(0, 0)	$0 + 0 \overset{?}{>} 10$ $0 > 10$ False	$0 + 3(0) \overset{?}{\leq} 24$ $0 + 0 \overset{?}{\leq} 24$ $0 \leq 24$ True	(0, 0) is a not a solution to the system because it does *not satisfy* both inequalities.

Example

1 Solve by graphing: $\begin{cases} x + y \le 3 \\ 2x - y < 3 \end{cases}$



Name two ordered pairs that are solutions and two that are not.

- Graph the first inequality.

  $x + y \le 3$

  $y \le -x + 3$ ← Solve the inequality for y.
  $m = -1,\ b = 3$

  The solution *includes* the boundary line $y = -x + 3$ and all points *below* the boundary line.

- Graph the second inequality.

  $2x - y < 3$

  $y > 2x - 3$ ← Solve the inequality for y.
  $m = 2,\ b = -3$

  Remember: Reverse the direction of the inequality symbol when multiplying by a negative number.

  The solution lies *above* the boundary line $y = 2x - 3$.

  The solution of the system of inequalities consists of the coordinates of all the ordered pairs in the area where the shaded regions *intersect*.

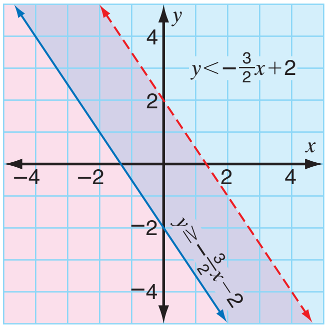
  $(-4, -2)$ and $(0, 3)$ are some examples of points that are solutions.
  $(4, 2)$ and $(0, -3)$ are some examples of points that are *not* solutions.

► Systems of linear equations that involve parallel lines are inconsistent. They do not contain a solution. This is *not* always true in a system of *linear inequalities*.

Examples

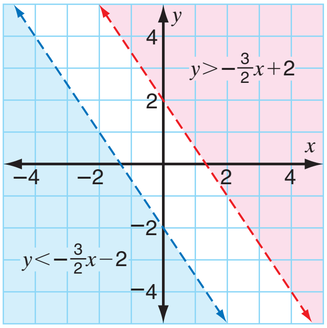
Graph the system of linear inequalities. Describe the solution sets.

1 $\begin{cases} y < -\frac{3}{2}x + 2 \\ y \ge -\frac{3}{2}x - 2 \end{cases}$



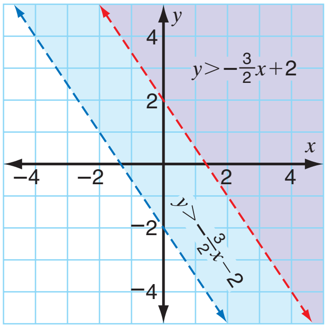
The solutions are all the points in the region *between* the parallel lines and *on the solid line*.

2 $\begin{cases} y > -\frac{3}{2}x + 2 \\ y < -\frac{3}{2}x - 2 \end{cases}$



The graphs of the two linear inequalities *do not overlap* or *intersect*. The system has *no solution*.

3 $\begin{cases} y > -\frac{3}{2}x + 2 \\ y > -\frac{3}{2}x - 2 \end{cases}$



The solutions of the system are the same as those of $y > -\frac{3}{2}x + 2$.

Continue Lesson ➡

► You can use systems of linear inequalities to solve verbal problems. You should consider whether fractions or negative numbers make sense as possible solutions.

For her birthday, Djana received a $75 gift card to an online bookstore. She wants to buy at least 6 books. If all books at this store cost either $8 or $5, write a system of inequalities that describe the situation. Then graph the system to show all possible solutions.

Let x = number of $8 books and y = number of $5 books.

Djana wants to buy *at least* 6 books.

$$x + y \geq 6$$

Djana can spend *no more than* $75.

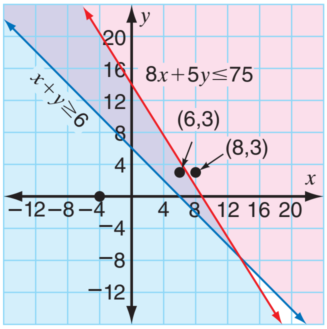
$$8x + 5y \leq 75$$

Solve by graphing: $\begin{cases} x + y \geq 6 \\ 8x + 5y \leq 75 \end{cases}$

- Graph: $x + y \geq 6$

$y \geq -x + 6$ ← Solve the inequality for y. $m = -1$, $b = 6$

The solution includes the boundary line $y = -x + 6$ *and* all the points above the line.



- Graph: $8x + 5y \leq 75$

$y \leq -\frac{8}{5}x + 15$ ← Solve the inequality for y. $m = -\frac{8}{5}$, $b = 15$

The solution *includes* the boundary line $y = -\frac{8}{5}x + 15$ *and* all the points *below* the line.

The solution of the system of inequalities consists of the coordinates of all the ordered pairs in the area where the shaded regions overlap (the purple region) and the ordered pairs on the part of the lines $y = -x + 6$ and $y = -\frac{8}{5}x + 15$ that are boundaries of the purple region.

- Check the solution.

In the context of the problem, the only reasonable solutions are those in Quadrant I. Since Djana cannot buy a negative or fractional number of books, the domain and range for this situation include *only positive integers*.

One solution is the ordered pair (6, 3). Djana could buy 6 books that cost $8 each and 3 books that cost $5 each. This solution *satisfies both conditions*, because Djana would buy at least 6 books and spend no more than $75.

$x + y \geq 6$
$6 + 3 \overset{?}{\geq} 6$
$9 \geq 6$ True

$8x + 5y \leq 75$
$8(6) + 5(3) \overset{?}{\leq} 75$
$63 \leq 75$ True

The ordered pair (8, 3) is *not* a solution. If Djana bought 8 books that cost $8 each and 3 books that cost $5 each, she would spend $79, and $79 is more than $75.

$x + y \geq 6$
$8 + 3 \overset{?}{\geq} 6$
$11 \geq 6$ True

$8x + 5y \leq 75$
$8(8) + 5(3) \overset{?}{\leq} 75$
$79 \leq 75$ False

Example

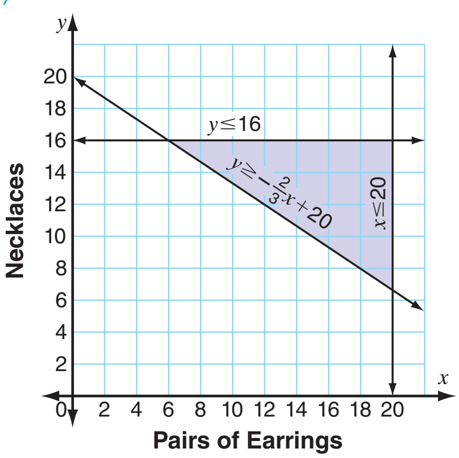
1 Sylvia's profit on her handmade jewelry is \$8 on a pair of earrings and \$12 on a necklace. She has the materials to make at most 20 pairs of earrings and 16 necklaces. From these, Sylvia wants to make a profit of at least \$240. Write a system of inequalities to represent the situation. Graph the system to show possible solutions.

Let x = the number of pairs of earrings.
Let y = the number of necklaces.

$$\begin{cases} 8x + 12y \geq 240 \\ x \leq 20 \\ y \leq 16 \end{cases}$$

- $8x + 12y \geq 240$ ← (earrings profit + necklace profit *is at least* \$240)
- $x \leq 20$ ← *at most* 20 pairs of earrings
- $y \leq 16$ ← *at most* 16 necklaces

Jewelry Profits



• Graph the first inequality.

$8x + 12y \geq 240$

$y \geq -\frac{2}{3}x + 20$ ← Solve the inequality for y. Use a solid line, and shade the region above the line.

• Graph the next two inequalities.

$x \leq 20$ ← The boundary line is a solid vertical line. Shade to the left of the line.

$y \leq 16$ ← The boundary line is a solid horizontal line. Shade below the line.

All possible combinations of earrings and necklaces that meet her requirements are represented by ordered pairs within the triangular region where the three inequalities intersect.

Two of the many possible combinations that meet Sylvia's requirements are:

(12, 14); 12 pairs of earrings, 14 necklaces
(20, 16); 20 pairs of earrings, 16 necklaces

Think
Only use combinations with whole-number coordinates. It would not make sense to sell a part of a necklace or earring.

Try These

Graph each system of inequalities.
Tell if the given ordered pair is a solution of the system.

1. $\begin{cases} y \geq 3x \\ x + y \leq -2 \end{cases}$ (0, −5)

2. $\begin{cases} x + 2y \leq 10 \\ 2x - y < 3 \end{cases}$ (4, −1)

3. $\begin{cases} -7 > x + y \\ 3x \geq -3y + 12 \end{cases}$ (−2, −2)

Graph to show all possible solutions to the problem.
Then name 3 ordered pairs that are solutions.

4. The Band Boosters hope to make at least \$400 at a fundraiser. They have 40 date books to sell at a profit of \$8 each, and 50 calendars to sell at a profit of \$5 each. How many of each could they sell to make at least \$400?

5. Discuss and Write Explain why the solutions to Exercise 4 have to be positive integers.

Go to PRACTICE BOOK Lesson 6-6 for exercise sets.