Problem-Solving Strategy:
Work Backward

Objective To solve problems using the strategy *Work Backward*

Problem I: Four college roommates—Ari, Ben, Joe, and Ty—baked oatmeal chews for their holiday trips home. The next morning, Ari woke up first and grabbed $\frac{1}{4}$ of the chews before he left. Ben then took $\frac{1}{4}$ of the chews that remained. Then Joe and Ty each took $\frac{1}{4}$ of the remaining chews in turn when leaving the dorm. If 81 chews still remained, how many had the roommates baked?

Read **Read to understand what is being asked.**

List the facts and restate the question.

Facts: Four college roommates baked oatmeal chews. One at a time, each took $\frac{1}{4}$ of the chews that remained.
After everyone had taken oatmeal chews, 81 remained.

Question: How many oatmeal chews had the roommates baked?

Plan **Select a strategy**

By using the strategy *Work Backward*, you can start from the amount that remains after everyone had taken oatmeal chews and figure out how much the roommates had baked.

Problem-Solving Strategies

1. Make a Drawing
2. Solve a Simpler Problem
3. Reason Logically
4. Consider Extreme Cases
5. **Work Backward**
6. Find a Pattern
7. Account for All Possibilities
8. Adopt a Different Point of View
9. Guess and Test
10. Organize Data

Solve **Apply the strategy.**

- There were 81 oatmeal chews left after Ty took his $\frac{1}{4}$. If t is the number of chews Ty found when he woke up, then $\frac{3}{4}t = 81$. Multiplying both sides by $\frac{4}{3}$ gives $t = 108$. There were 108 oatmeal chews when Ty got to them.

- If j is the number of chews Joe found when he woke up, then $\frac{3}{4}j = 108$. Multiplying both sides by $\frac{4}{3}$ gives $j = 144$. There were 144 oatmeal chews when Joe got to them.

- If b is the number of chews Ben found, then $\frac{3}{4}b = 144$. Multiplying both sides by $\frac{4}{3}$ gives $b = 192$. There were 192 oatmeal chews when Ben got to them.

- Finally, if a is the number of chews Ari found when he woke up, then $\frac{3}{4}a = 192$. Multiplying both sides by $\frac{4}{3}$ gives $a = 256$. So Ari found 256 chews.

So the roommates baked 256 oatmeal chews.

Check **Check to make sure your answer makes sense.**

Assume the roommates baked 256 oatmeal chews and re-enact the scenario.
Ari took $\frac{1}{4}$ of 256, or 64, oatmeal chews. So $(256 - 64)$, or 192, chews remained.
Ben took $\frac{1}{4}$ of 192, or 48, oatmeal chews. So $(192 - 48)$, or 144, chews remained.
Joe took $\frac{1}{4}$ of 144, or 36, oatmeal chews. So $(144 - 36)$, or 108, chews remained.
Ty took $\frac{1}{4}$ of 108, or 27, oatmeal chews. So $(108 - 27)$, or 81, chews remained.

The answer checks.

Problem 2: At dawn on the morning of July 1, Ruben is adrift on a raft 22 miles from shore. He can paddle 7 miles each day, but drifts back 2 miles each night while asleep. On what date will Ruben finally reach shore?

Read ▸ **Read to understand what is being asked.**

List the facts and restate the question.

Facts: Ruben has 22 miles to go.

He can advance 7 miles each day, but will lose 2 miles each night.

Question: How many days will it take for Ruben to advance 22 miles?

Plan ▸ **Select a strategy.**

Sometimes a problem can often be more easily solved by starting with the solution or destination, and then working backward toward the problem, filling in all necessary steps along the way. To accomplish this, you can use the strategy *Work Backward*.

Solve ▸ **Apply the strategy.**

Once Ruben gets within 7 miles of land, he can reach the shore in a day without losing 2 miles at night.

So "put aside" 7 of the 22 miles for the final days paddle. This leaves $(22 - 7)$, or 15, miles to consider.

Ruben's "net gain" each day is 5 miles (forward 7 miles and backward 2 miles). So he can cover $(3 \cdot 5)$, or 15, miles in exactly 3 days.

Therefore, Ruben can reach shore in $3 + 1$, or 4, days, which would be on the evening of July 4.

Check ▸ **Check to make sure your answer makes sense.**

As a check, run forward through the scenario.

July 1: Ruben wakes up 22 miles from shore; advances 7 miles; loses 2 miles. He gains only $(7 - 2)$, or 5, miles.

July 2: Ruben wakes up $(22 - 5)$, or 17, miles from shore; advances 7 miles; loses 2 miles. He gains another $(7 - 2)$, or 5, miles.

July 3: Ruben wakes up $(17 - 5)$, or 12, miles from shore; advances 7 miles; loses 2 miles. He gains another $(7 - 2)$, or 5, miles.

Ruben has gained a total of $(3 \cdot 5)$, or 15, miles in 3 days. He has $(22 - 15)$, or 7, miles to go.

July 4: Ruben wakes up 7 miles from shore; advances 7 miles during the day and reaches shore.

The answer checks.

Go to **PRACTICE BOOK** Lesson 6-9 for exercise sets.